Modelling as a Teaching-Learning Strategy

RODNEY C. BASSANEZI

The study of problems and real situations with the use of mathematics as its language for their comprehension, simplification, and resolution, aiming at a possible revision or modification of the object under study, is part of a process that has been named *mathematical modelling*.

In the modelling context, a mathematical model is almost always a system of equations or algebraic inequalities, differentials, integrals, etc., obtained through establishing relations among variables considered essential to the phenomenon under analysis.

The use of mathematical modelling has normally been limited to the research field where the target is to convert a systematic comprehension of empirical and concrete situations into mathematical models which are built and put into practice for a set of interrelated variables.

In terms of teaching, the use of modelling leads to the learning of mathematical contents connected to other forms of knowledge. Its application as a teaching-learning strategy, however, has seldom been used except in some isolated experiences.

Our job has been to try to connect teaching experience with the modelling perspective based on specific theoretical, philosophical, and methodological concerns We take into account the human resources, the interest shared by teacher, student, and community; social, political, and economical contexts, etc. We also look for the redemption of ethnomathematics, its interpretation and contribution at the level of mathematical systematization.

Working with mathematical modelling does not simply attempt to widen knowledge but to develop a particular way of thinking and acting: producing knowledge, putting together abstractions and formalizations, interconnected to phenomena and empirical processes considered as problematic situations.

The basic guidelines of this work are the following:

—To interrelate experimental and theoretical factors; that is, not to lose sight of the essence of the "mathematical attitude".

—To take into account the specific realities of every region and the students' interests, aiming at increased motivation and at an effective participation of the students in their communities or in a larger context in which they take part. It does not mean that it defends the popular thesis that "the science of a developing country must be regional" (which would be a mistake, since science must look for universal explanations through observable data, otherwise it is not science). Our intention is to encourage the students' concern with problems that surround them, even when scientific universal procedures are being adopted. For example, the analysis of the inclination of a trough used for gold sedimentation in a mining region, basically using trigonometry and notions of mechanics (as was the case during a course in Cuiabá-MT), or proposals for modifications in the cooling process for the preservation of apples through mathematical models which involve differential and mechanical equations in a community in Paraná which basically lives from the cultivation of this plant.

—To appreciate the human resources, explore and develop teachers' and students' skills, making them feel able to give the community their contribution and form socially active individuals In other words, try to follow Morley's proposal: "Join your people and love them. Learn with them, plan with them and serve them. Start with what they already know Build and teach them with what they have." Teaching should deepen students' knowledge, and for that they must be motivated to face mathematics not only as a science for its own sake but as an instrument for the understanding and possible modification of reality.

—To keep interdisciplinarity in mind—to associate mathematics to other sciences such as biology, physics, economics, history, etc.

Our teaching experience with mathematical modelling has shown the efficiency of this process in different situations:

1. *Regular courses:* courses with pre-established programs These courses use traditional approaches where the practice is almost always artificial and only a way of illustrating the theories taught When using modelling, the choice of topics to be analysed is related to the program of the course. This choice is oriented (never imposed) by the teacher and it is important that the students are involved in the process and feel motivated by the topics and problems raised The topics suggested by the students are the source of problems which stimulate the learning of new mathematical concepts in the search for their solutions. This is the ideal moment for the systematization of the applied concepts, and the analogy with other problematic situations makes the learning process more dynamic and involving.

For example, when working with fifth graders in a Junior High School—where the program consists of the introduction of numbers (natural, decimal, fractional), the usual linear, area, and volume measures, and elementary notions of geometry—the problematic situation ("building ordinary houses") was developed around this topic.

In the differential and integral calculus course for academics from the Food Technology course (at UNICAMP), the chosen topic was "potato plantation". All the program was developed from this and there was an active participation of the students in the teaching-learning process. The stated problem was how to plant potatoes in order to obtain maximum production. It is important to emphasise that in this course the approval rating was 100%, whereas when they were taught according to traditional rules and following sequenced content, only 70% of the students passed.

Similar efficient results were also obtained when we taught an advanced calculus course to students of engineering where the chosen work was to determine the dimensions of a swimming pool which was architectonically differentiated.

2. *Courses for bioscientists:* courses with an open program where the mathematical content is important when used to solve specific problems. We work this way in biomathematics courses, regularly offered to students preparing for master's and doctor's degrees in the area of the biological sciences (UNICAMP). In the method some classical models are analysed first, such as population dynamics, epidemiology, and ecology in general, so as to introduce the language and the arguments of differential calculus and linear programming. The mathematical prerequisites are worked on during the modelling process. Later, the students bring their own problems and dissertation topics in order to adjust them to the mathematical models. In this way we are able to analyse problematic situations such as "the flight of butterflies", "the generation of wasps", "the oxidation of blood in snakes", "the limitation of fishing", etc Some years ago, when we offered the first courses, they were an option for the students, but because of the impact caused by their papers, they are now compulsory in the biology post-graduate curriculum. In this way we have managed to cross over the strong disciplinary boundaries existing in our university, interrelating biology and mathematics Nowadays scientists from both areas are inclined to work together, and this fact led to the foundation, in 1991, of the Biomathematics area in the Department of Mathematics (IMECC-UNICAMP).

3 *Scientific introduction projects:* developed in small groups, or individually, by mathematics undergraduates, under the teacher's guidance. The goal is to learn how to use the mathematical modelling method, and the content focussed on depends essentially on the chosen topic. Initially we work with classical models so that students become familiar with the dynamics of the process The next step consists in the elaboration of alternative models adjusted to experimental or formulated data. Finally, new models are elaborated from analogies with other problematic situations after passing through a critical analysis of newly formulated hypotheses.

Some relevant models that were developed in this context concerned "production/fertilization", "ethanol acquisition", "biodigesters", "paper production", "foreign debt", etc.

The realization of these projects is more effective when the students are linked, or participate in bigger groups, doing research on some specific problem. This is the case of students from UNICAMP involved with the biomathematics group who work with topics such as "the evolution and treatment of malignant tumors", "the use of herbicides", "environmental pollution", etc.

4 *Adult capacitation courses:* supplementary courses to integrate adults into the formal educational system. The experience teaching the mathematics through modelling, in these courses, showed the approach to be very efficient. The adults start using mathematics as an instrument for the solution of daily problems, becoming also subjects in the teaching-learning process. An impressive example was the experimental course attached to Projecto Supletivo-PUC-CAMP (1989), when the students decided to "learn" mathematics following the "Project of economic adjustment" established by the Sarney government, basically focussing on the "question of wages" and the new currency: "cruzado novo". The problems related to the topics involved the study of the four operations, decimal numbers, proportionality, graphs, etc. At the end of the course, students presented their papers to a large audience. These papers not only led to an awareness of the potentiality of mathematics, but they also led students to acquire greater political consciousness (for example, they learned, through the language of calculus, which is supposed to be inaccessible to the worker, that the government plan was actually leading to a real decrease in salaries).

5 *Special programs for mathematics teachers:* the usual mathematical courses for teachers are very distant from social problems; they do not integrate mathematics with other forms of knowledge and follow obsolete programs without developing the necessary competence in the curriculum content As a consequence, there is a big gap between the pedagogic practices being used and the teacher's desire for effective participation in his own context.

The lack of interest, creativity, competence, and motivation are constant among maths teachers, who are forced to use inefficient textbooks which are almost always separated from the students' and teachers' socio-cultural context

In fact, the most serious problems faced by teachers are results of a social, economic, and political situation. The effects felt in our classrooms cannot be solved if one does not consider their origins

Without meaning to over simplify, we find that there seems to be a split between those who merely cross their arms, adhering to the general mediocrity, and those who, in one way or another, try to help the students' learning from within their reality. This apparently romantic, dreamy, or even quixotic position must be the result of critical examination of the program of the discipline, the real needs of the students, the regional context of the school, the students' prior formation and the teachers' background knowledge.

The lack of resources with an educational purpose is very common in our country, and in many others. This is why the educator's creativity, and especially his solidarity, is essential, because without them there is no education. Some teachers, aware of faults and distortions in their formation, feel the need for improvement, turning to refresher courses which are more frequently found in areas where university teaching is even more faulty.

We have been working as a teacher and coordinator (in the group of teachers from IMECC-UNICAMP) in many of these courses in the most diversified regions of the country. Once more, the teaching-learning strategy of mathematical modelling reveals itself as extremely efficient in valuing knowledge and encouraging the social participation of the professionals looking for these courses. We have opted to detail further the experiments of this working group as an illustration of the potentialities of teaching mathematics through modelling This work takes into account the fact that mathematics is an instrumental discipline, it must be developed using the questioning and uneasiness related to the environment the students live in. Our basic aim is, above all, to develop the capacity for analysing and interpreting data (empirical or qualitative), testing formulated hypotheses, creating models and verifying their efficiency in planning; in short, to give the students the conditions for understanding a phenomenon by participating in its transformation. That does not mean viewing mathematics as a purely utilitarian instrument. Mathematics must be seen as a dynamic science, characterised by generalizations and analogies. And a mathematical study may have an intrinsic importance, independently of its direct application. The notion that empiricism and immediate applicability in everyday life are what really matters is false and potentially frustrating. When a situation is analysed from the point of view of applied mathematics, the learning starts. From that point on, abstractions may follow unexpected ways, enabling the creation of new mathematical instruments and the formulation of new theories.

When working with mathematical modelling in adult capacitation or specialization programs, we start with definite aims:

—To enable teachers to change their concept of educational practice, helping them question some of the myths regarding the use of calculators, mathematical rigor, the sequence of contents, evaluation procedures, etc.

—To encourage innovative actions which arouse creativity.

—To value mathematical knowledge in a global context and its applicability in practical situations

The general program consists essentially of elementary mathematics, statistics, linear algebra, numerical analysis and computational methods, differential and integral calculus and differential equations This program is developed in three phases, mainly during school holidays, in about 360 hours. Each phase corresponds to three parts in which the subjects are interrelated and approached through problematic situations stemming from a topic and chosen by the students. The topics must be proposed by the student-teachers.

Phase 1: Ethnomathematics, statistics, and modelling 1

In this phase, possible topics of study related to production, economic, political and social conditions in the area, must be raised. The topics must be comprehensive enough to allow questioning from several directions

Participants visit many predetermined places according to the interest of each group. The goal of these visits is to see reality as a whole, trying to understand it from its peculiarities, or the other way round. The students are grouped according to a common interest and, once a topic is chosen, they return to the research field looking for new pieces of information collected through interviews, bibliographic references, and/or experiences lived and informed by the peculiar meaning of a particular culture Next the ethnological synthesis is worked out, interpreting the data collected in the research field.

Ethnology encompasses ethnoscience or, more specifically, in this course the ethnomathematics is one of the essential parts to be considered in the modelling process.

The ways of explaining and understanding reality and one's familiarity with it throughout the history of mankind make up a very rich source of questioning. Wine-barrel makers and cart-makers have their own methods and "mathematical schemata" which tell them how to insert the strips of wood or shape the rim of the wheels. The apiculturist "knows" when the honeycomb is filled up by simply observing the flight of the bees around the beehive. These pieces of knowledge are analysed and interpreted by the students through the use of mathematical modelling.

The purpose of the statistics component is, above all, to systematize data collection and analysis. In this part questionnaires are developed for interviews which are carried out with specific sampling methods. *A posteriori* analysis of the variables is done by testing hypotheses and adjusting curves.

Data collection is of great importance for the implementation of the modelling process

The greatest difficulty we have observed in adopting the modelling process in the courses we have been conducting is the breaking of the barrier posed by the traditional educational system, whose goal is almost always clearly outlined following a sequence of prerequisites and foreseeing definite results. In modelling, the beginning is merely a topic in which the mathematics to be used is still unknown. At this point we tell the beginners that when we do not have any idea what to do it is better "to count" or "to measure". This will always produce a table of data, and that may be the starting point of modelling. The simplest model may be obtained by adjusting data The formalization of a problem in mathematical terms is almost always the most difficult part of mathematical modelling and must be faced with a certain amount of experience as well as a scientific attitude and creativity (the study of interrelated and analogous models helps the acquisition of this background knowledge).

In Modelling 1 the first mathematical models are developed and they are almost always related to the contents of elementary mathematics (elementary and secondary schools). The problems initially proposed by the students are taken from research situations and are generally of short range effect. These problems have similar formulations to the ones found in textbooks, or are connected to the geometry of the research objects. Students also show a certain degree of inhibition in posing important questions, probably for fear of not being able to solve them.

Starting with these first problems, the ideas involved in them are simplified in search of generalizations and analogies with related situations. Their validity as finished models is questioned. In a general sense the problems proposed are rather simple and can be analytically solved by the use of proportionality, elementary geometry, progressions (arithmetic and geometric), straight line equations and parabolas, trigonometric relationships, combinatoric analysis, measurements, matrices, etc.

We can still analyse the way educational practice can be performed in elementary and secondary school classes using the strategy of modelling. In the following school term this technique must be tried out by the student-teachers in their classes. The results and difficulties may be analysed in the following phase of the refresher course.

It is interesting to notice that, from this point on, relevant changes related to the attitude of the student-teacher begin to occur: his motivation is activated, for he finally begins to be aware of the purpose of the content he had been teaching from the beginning of this career. He also realizes he is able to create new and much more meaningful and interesting problems, different from the ones found in textbooks. These changes can give the teacher a feeling of accomplishment.

Phase 2: Computational methods, numerical analysis and linear algebra

The objective of this phase is to approach the learning of initial topics in numerical analysis and linear algebra through the idea of mathematical models making use of computing The three subjects are developed simultaneously using the same initial themes as a basis

The formulation of new problems must follow a series of examples analysed by the teacher. The teacher must never clearly propose questions related to the research topics. All questioning must emerge from the group of students The instructor's role is to make the process dynamic and in the absence of questioning by the students find a way of leading them to the formulation of their own problems. The teacher must work as a monitor: he clarifies doubts and simply suggests an approach to the topic under study

Some of the topics developed in this phase are: the solving of equations, calculation of roots, curve sketching, the making of graphs, linear systems, vector spaces, eigen-values and eigen-vectors, linear programming, etc

It is a phase in which, besides emphasizing the practical aspects of teaching numerical analysis and linear algebra, both in the formulation of problems and in the quest for a method leading to an acceptable approximate solution, we also try to undermine the strong psychological resistance to the use of calculators in education.

Most problems are solved with the aid of calculators and, though the aim of the course is not "teaching computing", great emphasis is given to the use of computational software for micro-computers. The purpose is to display the resources that technology and the basic concepts of computing bring to education in mathematics.

Phase 3: Differential and integral calculus; differential equations

In order to make student-teachers familiar with the adequate language of calculus, the basic concepts of function, limit, differential and integral equations are introduced by means of previously elaborated models. Models are made by substituting mathematical language for the ordinary language Thus, terms like variation (stemming from increase or decrease) are translated as:

a *Simple variation* (the difference of the function between two points): $f(x_2) - f(x_1)$;

b. *Mean variation* (the average of the simple variation): $\dfrac{f(x_2)-f(x_1)}{x_2-x_1} = \dfrac{\Delta f}{\Delta x}$;

c *Relative variation*: $\dfrac{(f(x_2)-f(x_1))}{\Delta x f(x_1)} = \dfrac{\Delta f}{f\Delta x}$;

d *Instantaneous variation* (the limit of mean variation): $\lim\limits_{\Delta x \to 0} \dfrac{\Delta f}{\Delta x} = f'(x)$.

In this context, a given "law" or behavior of a phenomenon is analysed using the different mathematical meanings of variation.

Terms like "tendency", "stability", "equilibrium", etc, have their mathematical counterparts in the formulation of limits and asymptotes

Just as the mathematical model is made up it must also be interpreted in ordinary language. It then works as a bilingual dictionary: Portuguese-Mathematics and Mathematics-Portuguese The acquisition of the mathematical model presupposes the existence of a dictionary that interprets natural language through symbols and operations (models). The applicability of the model presupposes a decodification of its mathematical expression, so that the analysis of a situation can be made in both languages.

In this final phase of the course, mathematical modelling receives a more refined and complex treatment, searching for the "improvement" of models together with the critical interpretation of the obtained solution and its validity within the considered reality The whole process of modelling is re-examined and criticised. The elementary mathematical contents are valorized by transferring models created with the help of arguments from the differential and integral calculus, and from differential equations, to simpler models adequate for elementary and secondary school. For example, the Malthusian model of population growth, given by the equation:

$$\frac{dP}{dt} = \alpha P \quad \text{(instantaneous variation)}$$

can be analysed by an equation of differences:

$$\frac{\Delta P}{\Delta t} = \alpha P \quad \text{(average variation).}$$

The solution of the difference equation is simply obtained with elementary level arguments and both models represent the same law by Malthus: "the population growth is proportional to the population at each instant"

More than merely transposing models into languages, we were trying to demonstrate that the knowledge acquired in a university teaching course can and must be transferred to the elementary teaching of mathematics. It is only a question of knowing how to use the "bilingual dictionary".

The course may be completed with other subjects, depending on the interests of the group: geometry, general physics, history of science, etc.

The homogeneity of the group is also a responsibility of its members who, through discussion, try to reach the same level of understanding. Each group works on its own projects independently. The teacher works, most of the time, as a group monitor and proposes a joint meeting whenever deficiencies and ordinary questionings arise

At the end of each phase, each group presents the results of its research to the whole class, which may give suggestions for the improvement of the work. At the end of the course each group presents its work as if it were a "thesis defence", where the remaining participants must behave as an "examination board". There is then an exchange of experiences and criticisms, aimed at bettering each project and the course itself as a model of learning. Each group is evaluated on its performance, each student is evaluated by the members of his group as well as by auto-evaluation. The instructors evaluate the monograph presented by each group. These must include the models created in each part and the individual's work performed during the school term relative to the method

Throughout these years we have devoted ourselves to employing mathematical modelling in specialization courses, relying on a dynamic cohesive team who believe this teaching-learning style to be the potential generator of better qualified human resources. Mathematical modelling has itself been the object of research in mathematical education in master's and doctor's courses (UNESP-Rio Claro and FE-UNICAMP) and the demand from universities for specilization courses which include modelling has increased a lot in recent years

The topics chosen for research in the different courses are varied and unusual: horticulture, pig breeding, apiculture, piciculture, apple growing, paper manufacturing, children's games, mate growing, gold mining, public transport, soybean and wheat growing, soymilk production, Coca-Cola production, the foreign debt, frog raising, the Jesuit missions, wine making, paranoia, leisure, pottery, brickyard, cattle raising, supermarkets, textiles, elections, smoking, milk processing, wood reforestation, engineering, waste disposal, poultry, pluviometric rate, horse cart making, power supply to slums, sugar cane, etc. The diversity of topics is an example of the range of the program and of the development of the team members.

Some of the problems pointed out during refresher courses have ended up as research topics in the master's and doctor's programs at IMECC-UNICAMP; for example, the optimum chemical control of bacteria in paper manufacturing (Guarapuava-83); the epidemiology of leprosy and a study of jacare raising (Cáceres-92).

There is a wide gap between modelling and talking about its process.

In each experiment and the consequent discoveries there is a renewal of the belief that we can make use of the research methodology in the teaching-learning of mathematics with significant results

References

Andrews, J.G. & Mclone, G "What is Modeling", in *Mathematical modelling*, Butterworth, London 1979

Bassanezi, R.C. Modelagem como método de ensino de matemática, in *Educación Mat. en las Américas*, VII, 37, 1990, pp 130-155

Bunge, M *Tema e realidade* - Ed Perspectiva S.A., S Paulo, 1974

D'Ambrosio, U. Da realidade à ação, sumus Editorial-UNICAMP, S. Paulo, 1986

Freire, P. *Educação e mundança,* Ed Paz e Terra, R. Janeiro, 1983a

Gazetta, M. A modelagem como estratégia da aprendizagem da matemática... Dissertação de mestrado, UNESP, Rio Claro, 1989

Monteiro, A O ensino de matemática para adultos através da modelagem, Disertação de mestrado, UNESP, Rio Claro, 1991

Kapur, J N. "The art of teaching the art of mathematical modelling" *Int. J. Math Educ. Sc. Tech.*, 13, p. 185-192

Ferreira, E.S. "The Generic Principle and the Ethno-Mathematics", IMECC-UNICAMP, Campinas, 1988

It is the style of the Soul that comes out in the world of numbers, and the world of numbers includes something more than the science thereof.

Oswald Spengler
