

Difference, Cognition, and Mathematics Education*

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It seems apt, when our conference takes place in the so-called Third World, with participation of academics from the old European imperial powers and the newer ones north of the border, to discuss how understandings of difference might affect our analyses of cognition and mathematics education. To explore these issues, I want to discuss some implications of my research on cognitive development, class, and gender, the analysis of mathematics education as discursive practice and the approaches to practices of mathematics in formal and informal settings as in, for example, the work of Lave [1988], Scribner [1984], and Carraher [1988].

My aim is to speak generally, to open up a debate by examining some basic questions concerning how and why the issues that are raised are characterised in a particular way. In "The Mastery of Reason" [1988] I set out some ways in which I felt that post-structuralist theories might help us to understand the issues of context and transfer. I argued in that volume that there were some major problems with the way in which context and transfer were theorised, stemming from a view of context as something grafted onto a single model of cognitive development. I suggested that the problem lay within the theory itself and that simply adding on context was not enough. I proposed a theory of practices in which, instead of a unitary, fixed model of the human subject possessing skills in contexts, linked to models of learning and transfer, we might understand subjectivity itself as located in practices, examining the discursive and signifying methods through which a person becomes "subjected" in each practice. It is that analysis which I want to draw on here in suggesting that we might approach the issue of context and transfer differently in order to propose a theoretical account which deals adequately with the social and historical. To do this, I suggest, we need to abandon our view of the pre-given subject with skills and pre-social models of human cognition altogether. Such a view is, of course, not new, but I do want to suggest one theoretical pathway which might aid in this endeavour.

Let me begin by examining certain assumptions about childhood which are made in so-called First World theory. I vividly remember attempting to write some notes for a review of Lave's book sitting outside a cafe in the fashionable Coyoacan district of Mexico City. Of course there were the ubiquitous small children selling things: here artisianas, elsewhere Chiclets, and in some corners of the Third World, themselves. It would be easy to imagine a research project which aimed to examine the advanced calculating skills of such children of tender years and to compare this with school

performance. But what would it mean and what would we be doing it for? I also remember that, just as I was watching a small boy trying to sell toys, a little boy of about the same age, but clearly from an entirely different class, cycled past on his tricycle with his mother. It was this child who embodied the classical bourgeois idea of "child". Of course one could argue that these two children represented difference, or rather cultural differences and strengths, one in informal mathematics perhaps, the other in formal. But to approach the problem of difference in this way would be socially and politically a problem — for the two little boys are not "equal but different". They differ also in the type and extent of their exploitation and oppression. What I want to do here is to sketch out what might be the beginnings of a way of understanding psychologically and socially how difference is lived. When we concentrate solely on the cognitive aspects of performance we fail to engage with certain central aspects of the way in which oppression is experienced. That is, as in the above example, the child selling on the street is earning money which is likely to be a central plank in his family's survival. He has to learn to calculate then as if (to use an English phrase) "his life depended on it". Meanwhile we might point to the way in which such calculation is "low level" (a very common complaint in research on girls and mathematics, for example; see Walkerdine et al [1989] for a discussion) compared to the so-called "higher order reasoning" which the middle class boy might be able to perform. We might add that the first child is deprived and that this explains his poor performance at higher level tasks. But what I want to do here is to question this very line of argument. What *is* higher level and how can we make sense of an argument like this outside certain historical and social questions about the nature of mathematics and mathematics education itself? My claim is that if we begin to address these questions we set up our psychological arguments in a completely different way.

Let me cite another example. I remember sitting in a seaside cafe watching a mother and her sons negotiating the buying of drinks. The boys wanted cokes and the mother argued that these were too expensive and that they should have "a warming cup of tea". By contrast I watched a father and son sitting in a cafe in a park in central London making a game out of calculating change: "What if I bought . . .?", and so on. There was no economic necessity at stake here. Now, although it might look at first sight as though these two examples were similar, I would argue that there are some important differences. What does it mean that the father and

son in the park are constructing imaginary problems as opposed to the material problems faced by the mother trying to regulate her sons' consumption of expensive commodities? What is the effect of relative poverty and wealth on the way in which certain problems can be presented as "abstract" versus "concrete", or, as I would prefer to put it, problems of practical and material necessity versus problems of "symbolic control"? And what is the relationship between the classic concrete/abstract distinction and the one between a life in which it is materially necessary to calculate for survival and a life in which calculation can become a relatively theoretical exercise? Might calculation as a theoretical exercise have become the basis of a form of reasoning among imperial powers which depended for the accumulation of their capital on the exploitation of the newly discovered colonies? Do theoretical concepts come with wealth and what, if so, does this mean for economic and psychological theories of development and underdevelopment?

How do our ideas of "real mathematics" and of mathematical "truth" become incorporated into the "truth" about the human subject which is used in the regulation of the social? The "truth" of reason and reasoning, of the world as a book written in the language of mathematics, become important aspects of historically-specific regimes of truth. Carraher [1989] discusses this issue, but Foucault's idea of "truth" is useful because it allows us to link that mathematical "truth" with the "truths" in forms of management and government which aim to regulate the subject. So, for example, when Carraher tells us that Brazilian street children did not solve a problem by one-to-one correspondence we are left little option but to pathologise them since we have no other (socially and historically specific) theories on offer.

Historically, European accounting practices, for example, shifted in complexity when they introduced double-entry book-keeping. The money economy introduced in relation to mercantile capitalism a system of positive and negative integers. The transformations of "gold" into a commodity is accomplished by means of a number of transformations of signification in which the same signifier, "gold", becomes a different sign [Walkerdine, 1986, 1988]. Many people chart the fact that shifts within psychology and mathematics are related to particular social and political events and practices.

How do these issues get caught up in fantasy, the kind of fantasy which other classes and groups have of each other? Samantha, a white middle class English four-year-old is asking her mother why the window cleaner will get paid for cleaning the windows of her home. She is puzzling over the exchange relation between work and money and goods. Tizard and Hughes [1984] cite this example as showing the "power of the puzzling mind" of the four-year-old. Here they use a generic concept, and yet they point out that many of the working class four-year-olds in their study did not puzzle over the exchange relation. It was made painfully clear to the children: they were often told, for example, that they could not have certain things because money was scarce, and indeed that money was earned through labour. Why then do Tizard and Hughes assume that such a specific issue, which clearly relates to poverty and wealth, reflects a "general state of mind"? Are there indeed any general states of mind at all?

When poor children fantasise they may often dream of wealth, of fantasies of plenty in which they have, as one nursery school child put it in her domestic play, "chicken and bacon and steak". Or, like the seven-year-olds I cited in "The Mastery of Reason", they may find a shopping game that they are playing very pleasurable because, while they are supposed to be counting change and subtracting small amounts of money from ten pence, they actually laugh at the disjunction between prices and goods (a yacht for 2p for example) and pretend to be wealthy shoppers, put on middle class accents and generally have a good time. However, they do not get better at maths. The fantasies of poverty are wealth, the fantasies of mathematicians, according to Brian Rotman [1980], are of an ordered world. He calls mathematics "Reason's Dream", a dream in which "things once proved stay proved forever", a dream of order outside the confines of time and space. And the dream of developmental psychologists? Certainly for a specific configuration of developmental psychology and education that I shall go on to describe, there is a dream of a classroom as a natural environment with pain and oppression left outside the classroom door.

Mathematics and discursive practices

Theories of cognitive development, at least those stemming from the work of Piaget, have their roots in theories of evolution. They offer us generic theories of the development of a "species being" in interaction with an object world, ontogeny recapitulating phylogeny. Indeed, the development of the human species, at home with a mother, is often taken to be at the highest point of the evolutionary scale. In this view there is little room for history or for the social, except a social which is grafted on or which regulates rates of development according to a fixed sequence. This fixed sequence takes us from pre-logical to logico-mathematical reasoning which is at first concrete and then abstract. The assumed pinnacle of abstract reasoning is rarely if ever questioned. And yet of course it is precisely this which various groups are routinely accused of not being able to reach: girls, working class children, blacks, third world children, etc. And what I am putting forward is the germ of a suggestion that this simple sequence is itself a historical product of a certain world-view produced out of European models of mind at a moment in the development of European capitalism dependent on the colonisation and domination of the Other, held to be different and inferior. It was the European aristocratic and bourgeois male who was to become the model of a rationality founded upon a lifestyle in which economic necessity was not an issue and in which the domination of the Other was to become to a certain extent justified by a reading of difference as inferiority. That the position of those Others, the working classes and the colonised peoples, for example, was produced by their oppression and exploitation, their poverty, their appalling working conditions, letting a few of them slowly into education in order that European and colonial administration might eventually be undertaken by members of the dominated groups themselves, is a feature which is rarely brought into question when attempting to understand the production of psychological aspects of development.

Here I am referring to the way in which certain colonised peoples and members of the European working class were “educated out” for entry into the middle class, but that this meant effectively that regulators did not need to be applied by colonisers or the upper and middle classes because oppressed peoples could be taught to regulate the less educated members of their own groups. This made the whole thing more complex, and sedimented the idea that “the normal” was something that such peoples could aspire to and was something which they were not. Such concepts of normality and pathology could then become central planks of recognition and self-regulation that people took into themselves.

In order to set out some of the ways in which we might begin to understand this I want to outline very briefly the place of theories of cognitive development in British mathematics education and then go on to examine the idea, as set out in “The Mastery of Reason”, that we might understand mathematics education in terms of discursive practices. The idea of child development as a central plank of the early education of children in Europe generally, and in Britain in particular, has a long history, especially in relation to debates about childhood as a natural state associated with the idea of an education according to nature. The idea that reasoning is a natural phenomenon was to become the centrepiece of the new “scientific pedagogy”, using psychology, promoted from the end of the nineteenth century onward [Walkerdine, 1984]. The promotion of reasoning can be understood as part of what Foucault [1980] has described as the new modes of government, based as they were on the necessity for the production of scientific knowledge in the population, with particular emphasis on the new urban proletariat. Child Study Societies were set up in England around the turn of the century and many people followed Darwin’s example of monitoring the development of his infant son. The idea of mapping development was taken to mean that education could be scientifically controlled according to an idea of stages of development. There was an overwhelming emphasis on the idea of the norm and normality, through which the regulation of the population was to be assured [Rose, 1985]. In the early twentieth century, following the work of Itard and Seguin in France with Victor, the wild boy of Aveyron, which implied that humanity could be taught, Maria Montessori applied their methods to the education of children from Italian slums. It is here, then, that the idea that from the feral child to the child of the Other (working class, colonised) is just a short step, begins to take shape. Normal development can be monitored, humanness can be trained. The Other can be regulated by attempting to render him/her normal and by monitoring the pathology of development to try to put it right. The idea of development is of course presented as though it were a matter of “nature”; but this nature is very particular indeed. Many authors have noted that the model for reasoning normality is that of a white, bourgeois male [e.g. Sayers, 1982; Le Doeuff, 1979]. It should be noted, therefore, that the idea of normal development carries with it a very oppressive model of the natural, in which the idea of a normal course of development is used as a regulative device. While such ideas were incorporated into pedagogic experiments in the early decades of the twentieth

century, it was not until the post-war period that they really began to come into their own in State education. The climate was ready in the 1930s, but the war intervened and it was not until the expansive years of the 1960s that “child-centredness”, as it became known, became incorporated in a large way in curriculum development. Mathematics education was rather slower than other curriculum areas to take up these ideas, but there were two reports in the 1950s and 60s, one by the Mathematical Association [1956] and one by the Schools Council [1965] which advocated the “new pedagogy”. In doing so, the central idea of mathematics as reason became enshrined within the curriculum. This was widely interpreted to mean that logico-mathematical principles could be used to code all activities, and this became translated into a kind of commonsense understanding in which everything became potentially mathematics. There was an inherent confusion because it was assumed that children were unable to recognise that mathematics is everywhere. In this analysis, representation was grafted on to an unproblematic base of action. In “The Mastery of Reason” I challenged these central notions, arguing that “mathematical” signs are produced within specific practices and that these practices are always discursive.

Let me give some examples. I analysed the way in which so-called mathematical signifiers, such as “more” and “less”, were produced within domestic settings in the homes of a sample of thirty four-year-old girls and their mothers [Tizard and Hughes, 1984; Walkerdine and Lucey, 1989]. While it is commonly assumed in early education that “less” is more complex than “more” and that the two form a pair, a contrastive opposition, describing the comparison of quantities, analysis of the mother-daughter exchanges revealed that although there were plenty of examples of the comparison of quantities, these were not described using the word pair “more/less”, and that while “less” was not used at all, “more” was used frequently, but in the context of the regulation by the mothers of their daughters’ consumption of commodities. For example, a mother might tell her daughter that she could have no more of a particularly expensive commodity or that she could not have more food until she had eaten what was on her plate. The contrastive pair here was not “more/less” but something more like “more/no more”. It will come as no surprise that such terms were used more frequently by mothers in working class families, so that such little girls would be more likely to understand their mothers as more regulative and to have very strong negative associations with the term “more”. (In Walkerdine [1990] I cite an example from my own history: my mother’s use of the phrase “much wants more”.)

What then will such children make of the use of the term “more” to describe the comparison of quantities in early mathematics? I argue that this may be the same signifier as in the practices of the home, but it is not the same sign. This difference is crucial for it suggests a more complex issue than existing practice might suggest. I argue further that such signifiers are made to signify when united with a signified within a particular practice, from which they take their meaning. Such practices are discursively regulated with the participants positioned in particular ways. The idea of the production of

mathematical signs within practices renders them at once both socially and historically specific and links them to the non-rational non-cognitive axis by the use of Lacan's [1977] transformation of Freud's theory of unconscious chains of association into chains of signifiers.

I further analysed the ways in which domestic practices are discursively different from or similar to school mathematical ones. Although the analysis of "mathematics is everywhere" stresses the similarity between practices, such an analysis glosses over important differences. This discourse stresses the idea of transfer and the sense that all experiences can be analysed logico-mathematically. My analysis stresses why and how practices are made to signify and suggests that the relation between family and school practices is far more complex than is suggested by the notion of doing mathematical examples in familiar contexts. I examined examples of mothers and daughters cooking together and asked when and how cooking could be said to have become mathematics. Certain quite specific discursive transformations took place when cooking became mathematics. In every case the discourse moved away from the product of the task, something to be cooked, towards a mathematical string, with a particular linguistic form, in which all external reference was removed from the string itself (as for example in the string $2 + 3 = 5$). I argued that cooking could not be said to *be* mathematics, only to act as a foil for it, until this transformation had occurred. This concentration on the mathematical string for its own sake, moving away from a product, is typical of the mathematical tasks which I observed in early education. Indeed, the analysis of the shopping game to which I referred earlier, makes it clear that one of the problems for the group of seven-year-olds was that this game was represented as shopping but that the regulation of the game was quite distinct from that of shopping practices. For example, each child had to choose a card with an item to be bought and an amount of money less than ten pence. They had to work out what change from ten pence they would get if they bought the item using plastic coins and to record the calculation on paper. As I have explained, the group found the disjunction between the game prices and "real" prices the basis for considerable humour and fantasy. They also had a fresh ten pence piece each time so that their money never decreased as it would have in real shopping, and their end-product was a calculation written on a piece of paper and not a number of purchases. In other words there was absolutely no exchange. Now, this is the issue that I referred to right at the beginning of the paper: The calculation has apparently become abstracted from its insertion in everyday practices. Yet to use the term abstracted is misleading, for the new calculation exists as a discursive relation in a new set of practices, namely those of school mathematics, with its own modes of regulation and subjection. The child moves from the position of a shopper to that of a student, for example. What I am trying to establish is that this move is not best described as a shift from concrete to abstract but as a move from one discursive practice to another. Secondly, what comes to be valorised as a higher order activity might have everything to do with attempts to regulate and control through reason in a social order which finds its norm in a bourgeois subject who does not need to calculate to survive. Thirdly, the new discursive prac-

tice of school mathematics has its own mode of regulation and subjectification. By this I mean that each child becomes positioned as a subject in a new way. That way may be similar to or different from the patterns of subjectification in other practices, but evidence suggests that for oppressed groups the patterns are substantially different. This may have important affective consequences. All of this suggests that the idea of children and adults possessing different skills in different contexts can be shown in a new light. Scribner and Carraher's subjects, for example, are not bourgeois subjects: they are oppressed and exploited groups — working class men in the USA and children from the Brazilian lumpen proletariat.

I should like to end, then, by attempting to exemplify the ways in which oppressed subjects may live the different positioning from practice to practice. This disturbs the cosy picture of the rational unitary subject [Henriques *et al.*, 1984], the "natural child" of developmental psychology, and substitutes an account which is specific to time and place and against which Reason's Dream looks like one more colonial fantasy.

Splitting the difference

How do children manage the transition from one practice to another? Although it is common in psychological accounts, especially from the 1960s and 70s, to suggest that it is good mothering that prepares children for success in school [see Walkerdine and Lucey, 1989 for a review] such accounts are problematic in that they imply that the problems experienced by children from oppressed groups are the result of inadequate mothering. Such accounts deny the complexity of the pain of moving from subjectification in one practice to another which appears to have a completely different set of rules and expectations. How might children from oppressed groups cope with and defend themselves against the pain? Althusser [1971] in his famous Ideological State Apparatuses paper used Lacan's theory of the mirror stage to argue that schools interpellated children as subjects, creating imaginary identities for them. Lacan used the idea of the mirror to suggest that the child's first view of itself as whole and unitary was the first ideological illusion. Now, while the identity created by the school may well be a fiction, it has powerful effects. While Lacan may be quite correct in asserting the illusory nature of the idea of a coherent identity, it is undoubtedly the case that subjects from oppressed groups experience more keenly a disabling sense of fragmentation [Mama, 1987].

The title of this section refers to the psychoanalytic term "splitting", which is one of the mechanisms of defense against extreme anxiety. While Freud [1951] and Melanie Klein [1975] use this term in rather different ways, both refer to the way in which the unconscious defends itself [see Walkerdine, 1985; Hollway, 1984; and Urwin, 1987 for further discussion]. Although on the surface some children may appear to be dealing with the transition from one practice to another in a detached manner, it is precisely this detachment which psychoanalytic accounts suggest is a key to extreme distress. Super-rationality may be a defense against extreme anxiety. One of the six-year-old girls in a study which I conducted presented in class the appearance of extreme stupidity. She could not follow a simple instruction

and was extravagantly vague. It later became clear that her vagueness was her best defense, the way in which she routinely cut off from the fact that her mother was being systematically physically abused by her father. Her violent feelings only emerged in an incident in which she had broken the heads off some dolls in the Wendy House

Patsy is a working class girl who at the age of four was part of the Tizard and Hughes [1984] study. We [Walkerdine and Lucey, 1989] saw her at ten. At four she was, like many other girls in the study, having difficulty with coming to terms with being a "big girl". However, if her mother positioned her as a "clever girl" she was willing and able to carry out certain tasks. The positioning as her mother's clever girl was important. She also scored high on an IQ test. However, at ten she was certainly not positioned by her teacher as "clever". Rather the teacher categorised her as "nowhere near as bright as the rest (of the class)". She said that Patsy resorted to infantile behaviour and that basically she had no saving graces. How come this "clever" little girl became stupid and infantile at ten?

It is a shocking fact that three other working class girls in the sample who gained high IQ scores at four were also regarded as stupid at ten and they all, like Patsy, positioned themselves as victims. While many girls mentioned the violence of others, especially boys, these girls saw themselves as the target of that violence. Using the psychoanalytic discourse which I have discussed it is possible to see this as a defensive response to unendurable pain. What if Patsy and these other girls felt frightened in an alien world that they did not understand and which did not understand them? They could not easily unleash their anger against those who they needed desperately to call them "clever", to make them feel safe and at home. To project their violent emotions into others and present themselves as victims as reminiscent of some of the symptoms displayed by colonised peoples as described by the psychiatrist Franz Fanon [1967] when talking about the Algerian War. Sometimes to learn to split is to learn to survive and to long to be loved in an alien world in which it is all too easy to be rejected. Another defense, of course, is to do the rejecting first, so as to make the pain of failure more bearable. Sociologists have tended to describe such strategies as anti-school resistance [e.g. Willis, 1977].

For all of these children, crossing the boundaries from one practice to another cannot be easy. In Walkerdine *et al* [1989] we pointed out that no girls cross the boundary from home to school as an easy transition from dependency to autonomy. When girls enter school they are classified, categorised. The readings of their behaviours and performance are highly gender-specific. We presented ample evidence to support the view that even when girls displayed the characteristics valorised in boys this did not mean they were judged as being successful. Often, precisely these designations rendered them pathological when viewed in relation to femininity. We argued that it was necessary to understand how high-performing girls came to be designated as "only hard-working" when poorly-achieving boys could be understood as "bright" even though they presented little evidence of high attainment. Poorly-achieving girls in the study, quite simply, were *never* designated bright [see for example, Walkerdine *et al*, 1989, page 102].

In other words we presented a whole system of subjectifications through which girls are judged. That these subjectifications have little empirical foundation in relation to the girls' performance further points to the importance of the sense that some fiction is being created to account for what it is necessary to prove time and time again: the inferiority of the Other. The Other constantly threatens the dominant group and no end of fantasies and fictions are employed to position the oppressed subject as Other, pathological. We argued further that since Reason has to be understood as the possession of "man", there will always be a push to prove Otherness as "lack". It is indeed the paranoias of the powerful that are at stake here: the fear that the oppressed might be able to take away their position of dominance. It is our contention that this dominance has to be assured by a number of social and psychic strategies for constituting the oppressed groups as Other and so pathologising them.

Such issues bring us back full circle to the pathologisation of difference. It is my contention that any psychological approach to the issue of difference and mathematical performance must deal with the complex psychic issues raised above. The fantasies of the coloniser are inscribed in the regulation of colonial subjects [Bhabha, 1984]: they become the "truths" through which development and performance are understood. Those fantasies and the attempts at regulation are inscribed in the very history of the insertion of theories of reason and reasoning into mathematics education, and wherever we find the Other, the working class, the peasant, the black, the girl, there we find claims of the proof of abnormality, of irrationality. My argument finally is then that in order to address these issues properly we need to construct accounts which move away from the stagewise progressions of most First World developmental models to an understanding of development as specific to social and historical circumstances. Only then, I suggest, will we be able to engage with oppression as something other than individual pathology.

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