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The image on the cover is taken from Figure 4, page 31, of the Danielson article.

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TEACHING MATHEMATICS

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I very much enjoyed reading the stimulating short communications in FLM **24**(3). Presented consecutively over a dozen pages, they triggered a number of personal associations, which linked them together for me in perhaps unintended ways.

To begin with, I was reminded that the original editor, David Wheeler, had always hoped for more such contributions, for more interactive debate. Then I realised that next year was going to be the 25th anniversary of his founding of the journal. Its attractively modest, lower-case title not only indicated a shift of attention at the time to the learning of mathematics, but also contained a quiet, coded reference to Caleb Gattegno's *For the teaching of mathematics*, a collection of his articles written in the nineteen-fifties [1]. Gattegno had worked in England in the fifties and had influenced a generation of mathematics teachers. He provided some of the links I found between the communications.

One such link was algebraic. Abraham Arcavi and Barbara Dougherty discussed ways of teaching linear equations through visualisation, and the value of this in general was emphasised by István Lénárt. I was reminded, however, of an alternative approach to the solution of equations, adapted by Gattegno from a traditional (at least in UK schools) classroom game. An early reference to this is in Gattegno's chapter, *La pédagogie des mathématiques*, in a 1955 collection of articles by various authors [2].

> Le maître demande à A de penser un nombre inférieur à 10 par example, à B de fournir un nombre egalement moindre que 10, et à C ce que A doit faire de ces deux nombres; le résultat de calcul énoncé par A, on demande quel est son premier nombre.

But the crucial point is that the game develops with an emphasis on operations and their inverses, and "au bout de deux leçons",

> Des examples tels que
>
> $$\frac{((R - S)/T + U) \times H + M}{J} = G \quad \text{trouver R}$$
>
> sont résolu en quelques secondes et paraissent de loin plus faciles que ceux de l'arithmétique.

That this can be done, and indeed in two lessons, is easily verified.

In his communication, Lénárt asked what algebra might mean for general education. For Gattegno, algebra is much more than a school subject, it is

> another name for the awareness of the dynamics involved in the transformation of mental structure available to us all.

From this point of view, the rich historical development of algebra, discussed by Arcavi, may be augmented by an awareness that this has been a mathematisation of an operational fluency that people acquired for themselves as babies. According to Gattegno,

> there is great advantage in extending the meaning of the word 'algebra' so as to be able to stress that it is concerned mainly with mental dynamics, rather than only with mathematical operations.

Thus, Gattegno drew attention to such facts as that I can move my eyes to the left and then back, that I can fold my fingers and then unfold them, that I can link a sequence of transformations *tap – pat – apt* with the sequence *top – pot – opt*, and so on. Of course, I may not know that I know these things. Alf Coles reported, in his communication, that when he started teaching a regular refrain from his students included 'I can't do algebra' – they can but do not yet know they can. I was reminded of a radio talk Wheeler gave in 1962 in which he suggested that

> all our efforts to justify the way we have been teaching mathematics look pretty silly when we realise that inside every bored schoolchild there is a mathematician screaming to get out.

Coles discussed the issue in terms of Bateson's levels of learning. I link these myself with Gattegno's treatment of levels of awareness. I may become aware that I can do algebra, but then, in a possible further step, I may become aware of this awareness. In so doing I become aware that there are all sorts of other things I say I cannot do, which I can also address. As Coles implies, there is more to the learning of algebra than being able to solve linear equations.

When re-reading Gattegno's chapter on pedagogy, I recalled Tom Kieren's reference to "the political significance of our work", for this was much in the minds of Gattegno and his co-authors, who were all involved in one way or another with the so-called modern mathematics movement in the idealistic, post-war mood of the nineteen-fifties. The psychologist Piaget, the logician Beth, and the mathematicians, Dieudonné, Lichnerowitz and Choquet, shared Gattegno's vision of a humanistic, student-centred, mathematics education. In an exhausted, war-weary Europe, curriculum reform at that time involved much more than the now usually assumed emphasis on axiomatics.

In his chapter, Gattegno distinguished three issues: the psychology of learning, the nature of mathematics, and the teaching of it in a social context. Later, he came to address the latter concern in a typically individual way. In a 1960 talk on Canadian television on mathematics and the needs of society, he suggested that we had to learn to think in a complex

way about complex questions. For him, mathematics was a way of meeting the aim that "each of us be prepared to meet the unknown rather than ready-made situations". Twenty years later, he was still offering the same challenge in the context of a short article on algebra:

> As long as we stick to a traditional interpretation of algebra (now known to be false because so limited) we cannot educate our youth as it needs to be in order to face the future with fewer dangers.

In her account of ICME-10, Fulvia Furinghetti noted that there was "a lack of the voices of teachers". She also referred to a lecture on the work of Klein and Freudenthal: both were no doubt excellent university teachers and influential figures in mathematics and mathematics education, but it would be fair to say that neither said much about the actual act of teaching in schools – about what people do with themselves when they work with classes of children. But then not many of us do. It is perhaps not surprising that few step in to where angels fear to tread – it is always safer to analyse the curriculum, or to propose and develop new ways of delivering it.

In discussing the success of one of his students, Coles modestly – and rightly – emphasises that she was the one that caused the change in herself. But his readers can see that the point is that he had paid attention to her. And it is this *attention* that is at the heart of the matter. It is always implicit in the accounts teachers give of their own classrooms, and it often becomes explicit when people are able to observe and discuss a lesson given by another. It was Gattegno's distinction as a mathematics educator that he was prepared to teach children of any age or ability, anywhere, at anytime, and in whatever numbers, and to do so publicly in front of other teachers. And it was the way he gave attention that was so often the focus of discussion: it was negative for some observers, positive for others, but in either case it was attention that was the issue.

In anticipating a 25th anniversary, I looked back (in the spirit of Furinghetti's plea for attention to the history of the teaching and learning of mathematics) to the time when Wheeler was a young teacher in the UK and an active member of the Association of Teachers of Mathematics (ATM), founded by Gattegno.

The latter's influence meant that meetings of the association always included a so-called 'demonstration lesson', where one member would teach a class of children observed by the others. I recall some of the occasions when I had been among the observers of a lesson given by Wheeler. Some brief notes that he provided for one such occasion in the early sixties ended in a style that his many friends might fondly recognise:

> If there is any value in this kind of activity (which will be for you to say afterwards), it clearly has a great deal of relevance to the classroom, because it is there that this permanent "laboratory" exists. (Question: how can we use this laboratory more systematically?)
>
> If this all sounds too solemn and pretentious, this is because it is difficult to talk about. In practice it can be most entertaining. I hope it will be on this occasion.

It was indeed entertaining, but also highly rewarding. And ever since, his authority has been bound up for me with a vivid sense of his attentive presence in a classroom. Despite its title, I hope the journal he so brilliantly founded continues to be always for the teaching, as well as the learning, of mathematics. May it thrive in the next 25 years.

Notes

[1] Three volumes, published in 1963 (by Educational Explorers, Reading UK): vol. 1 with articles on pedagogy, vol. 2 on psychology and on use of films, vol. 3 on elementary mathematics and the use of Cuisenaire rods.
[2] The first publication of the Commission Internationale pour l'Etude et l'Amélioration de l'Enseignement des Mathématiques, which was *L'enseignement des mathématiques* (Delachaux et Niestlé, 1955 – second edition, 1960), written by six founding members (there is an excellent history – up to 1985 – of this Commission by Lucienne Félix).

I have my own reservations about the word pedagogy, but I want to insist on it, temporarily at least, to indicate that teaching is a serious enough business to be worth investigating and developing with as much attention to detail, sensitivity to phenomena, and intellectual rigour, as any other human or social science. I don't know whether you can accept this; clearly our society as a whole does not. The folklore of our society about teaching suggests that (1) teaching is very straightforward: all that is required is "to know your subject", and that (2) when teaching turns out to be not that simple, it requires someone who is "a born teacher". (I emphasise that these folk-beliefs are widely-shared.) Now what could be more destructive of the possibility of pedagogy, what could devalue teaching more, than beliefs that there's nothing to teaching mathematics other than knowing mathematics and having the good fortune to be naturally endowed with some undetermined skills? It isn't surprising, viewed from this perspective, if mathematics is not taught very well, if mathematics teachers are not trained very effectively and are not greatly respected by the general public. Even many of those in the profession, who know that the folk-beliefs are false, are not confident that they can exactly tell what the teacher of mathematics needs to know other than mathematics, or how these know-hows can be acquired if they are not already possessed. The problem approaches the circular: pedagogy doesn't exist because there is nothing to show for it; there is little to show for it because not enough people believe in its potential existence.

(David Wheeler, talk given to the Canadian Society for the History and Philosophy of Mathematics, 1985)
