Zed lived an adventurous life that included fieldwork spanning over 50 years with school children in the UK, Italy, Australia, Brazil, Canada, Papua New Guinea and the United States. His body of work will remain an inspiration for generations of mathematics educators who place mathematics at the center of mathematics education.

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My students deserve better

PETER TAYLOR

In my third-year course, *Mathematical Explorations,* designed for future high school mathematics teachers, I had my students submit journals this year. Ranging from 4 to 40 pages, they discussed the problems we had worked on and reflected on their own learning. I see now that some of the real struggles they (and therefore I) seemed to be having, especially during the first half of the course, worked out pretty well for most of them. I also see that there are some things that might be changed. For example at the early stages it was hard to get much participation; I have to rethink my expectations.

My experience with this class has given me new insights into my two large first-year courses: calculus and linear algebra. I've been thinking about those courses over the past few years, trying different kinds of problems, different ways of interacting with the class, and though things seem to be working pretty well, I've always felt that there was something fundamentally amiss. My main purpose here is to think about ways in which those courses could be more like my third-year course.

In my third-year course, the problems we work with involve mathematics that most of the students have seen before but they are challenging in the sense that one has to play quite a bit in order to begin to see what sort of strategies might work. They are chosen for their power to deepen the students' understanding of the ideas and to lead them to a new appreciation of mathematical structure. According to the students, the main difference between this course and others they have taken lies in its pace (slower) and thrust (deeper and wider). (Aren't deeper and wider opposites? Not really—lateral connections reveal new structural properties.) The objective is as much to give the students a chance to confront and develop their learning skills as to deepen their mathematical understanding:

> As I reflect on my learning in math throughout my university career and in this course, I find that [...] I haven't "done" or "learned" math since high school; I have memorized and regurgitated the knowledge of my professors in hopes of getting good grades and finishing courses. The knowledge that I retained from all of this felt minimal, and it probably was, but this class helped me to do and learn math for real again. I realized that I did learn in my first three years in university but I didn't know how to apply my knowledge. Math became a daunting, scary mountain that I couldn't climb because I had forgotten how to apply what I know and really do math. But MATH 382 reminded me that math can be fun, and reminded me how to really DO math. (Kirsten)

What I discover from the students' journals is that this experience of digging deeply, of taking things apart to see how they work and then putting them back together, of constructing simple concrete examples as a way of playing with ideas, was new to most of the students. Remarkably enough, after 14 years of formal learning, they have spent almost no time in play.

That's not quite right. A number of our students, perhaps a quarter, have certainly spent a lot of time in their lives in mathematical play. When kids are young, they bend the rules and twist things into the wrong shape just to see what happens—that's their job as kids. But later on this natural behaviour seems to get schooled out of many of them, and they increasingly adopt safe strategies which seem to offer short-term gain. Only a few resist these temptations and keep right on playing. Who knows what makes the difference? Perhaps some early success, a key learning experience, an unusual teacher, or just a natural appetite for risk-taking. In any event such students do well in mathematics partly because they develop powerful learning strategies, but also simply because they've put in the time because they find playing with mathematics more fun than texting. I believe that our current undergraduate program serves these students very well.

It's the remaining, say, 75% of our students that I am interested in here. I have no doubt that these students have the capacity for serious play, but somehow, in their early years, they abandoned it, and it's hard for them to get started again. I know that there is considerable work being done on the question of how to get more students to keep on with that mathematical play. The question I am asking here, however, is: given the students I have now, what should I be doing in my large first-year classes?

> I had to think about it right then and there in the lecture when usually I'm just trying to keep up with the professor's handwriting, hardly listening to what they are saying. (Ashna)

The answer seems clear enough to me. I need to *teach less and discover more.* Rather than deliver the *product of mathematical thought,* engage them in the *process of mathematical thinking* (quoted from a paper by Asia Matthews). I don't mean to disparage "the product of mathematical thought" (more simply described as "mathematics"). It's real knowledge, particularly in a world in which much of what passes as knowledge is suspect. It's solid and eternal and has beauty and structure to die for. Nothing else in the knowledge world comes close to touching it. But my primary job as teacher is *not* to convey knowledge (narrowly interpreted), but to interpret, to transform, to enable, to bring to life.

I think that there are two components to this program. The first is to find a set of discovery problems that (if you like) cuts a natural path through the absurdly fat text-books that we far too often make our students buy in first year. And the second is to find a way to deliver those problems in a large class. I've been working on the first component for many years and it's coming along fine. The second component is more challenging and my experiments over the past years have had mixed success.

I don't use clickers, but I have often put out a small problem for the students to "pair and share" or simply think on their own. Sometimes this works quite well, but more often I feel, as I'm wandering around the room, that most of the students are sitting empty without much to think or say. When I start up again and ask for comments, the same few hands always go up. I'm thinking that this form of the consultative process doesn't work so well in mathematics.

> My first thought when Professor Taylor mentioned his struggles with his first-year students was that he definitely would have struggled to get me to engage when I was in first year […] if my professor asked me to collaborate with other students in class, I would probably just sit there and not contribute. (Michael)

Thinking back to my own student days I know that I would never have wanted to talk about a problem with a neighbour until I was ready to do so and that readiness can seldom be rushed. In fact I completely avoid discussing a problem until I have managed to centre it in my mind and assemble the necessary pieces beside it, and that takes time. As a student, what I wanted most from a lecture was a good story.

And that brings me to "discovery learning." Lately it's been in the news (not always favourably) and usually misunderstood. The most extreme misconception is that it expects students on their own to rediscover hundreds of years of hard-won knowledge. For me, discovery learning is best described as a style of communication. It begins with a problem or more generally with a narrative or story that is "open" in the sense that it invites exploration and further development. A lot of my curriculum work has involved the construction of such stories.

> I think my favourite part about this problem is the way it was framed; it makes it much more interesting and fun to solve. (Kirsten)

> I believe there is a lot of value in being able to work out a problem intuitively before exploiting any existing theorems or results, which seemed to be a theme that was emphasized throughout the course. (Makenna)

A story is a wonderful way of posing a dilemma, floating a paradox, setting up a quest. But then the action has to roll, the dilemma has to spin out and unwind. How is all that to happen in a large first-year class? Whitehead talked about this:

> In my own work at universities, I have been much struck by the paralysis of thought induced in pupils by the aimless accumulation of precise knowledge, inert and unutilized. It should be the chief aim of a university professor to exhibit himself in his own true character—that is, as an ignorant man, thinking, actively utilizing his small share of knowledge. (Whitehead, 1967/1929, p. 37)

We learn from example. A good example can be abstracted and retooled to fit onto a new problem. This applies also to learning how to learn. We learn about blocks and marbles by watching other kids play with them. We learn about playing with ideas by following the thoughts of a teacher.

> Other loosely related problems may have to be solved, to generate experience and insight. (Peiling)

In fact "playing with ideas" is not what it might seem. Ideas are abstract and play is concrete. We discover things by mucking about, by getting our hands around things, shapes, numbers, equations, concrete things as simple as we can make them without losing the piece of structure that has bedeviled us. Our first-year students can learn a lot simply by watching us reinvent examples, by witnessing that hands-on analogical process at work.

> This was my favorite problem because it shows math is very interesting and math is MAGIC. I cannot believe that math concept can help to construct such amazing pictures. The usual math problems I met are talking about proofs, derivatives and calculations. But this one can really trigger me to think something deeper—like math in my body. (Shuming)

So for me, discovery learning emerges when the student has wrestled with the problem in the tutorial or taken it home. But to enable that, to set it up, what's needed is what might be called discovery *teaching* and that's what Whitehead was describing: a reflective playing with an object of beauty.

> The amount of structure in this problem is truly amazing. (Jacob)

So that's my game plan for my first-year class this coming semester. Take a problem, a good problem with some marks of sophistication, and before their eyes, "actively utilizing my small share of knowledge," track it down, wrestle with it, bring it to the ground, and then stand back to let it rise up again, transparent with its inner structure displayed. Well, that's a plan; it puts a lot on the shoulders of first-year students. But if they are able to rise to the occasion, I will promise to organize the kind of technical and conceptual support they will need.

Postscript

I find a yearning for freedom in some of what my students have written: the freedom that comes from being in control and maybe even being a bit out of control. Anyway, the freedom of having your own hands on the wheel and your own foot on the accelerator. As a teacher, I also find myself looking for that kind of freedom and I know that other teachers do too. The hard thing about mathematics teaching, except at the advanced level, is that so much of the mathematics we teach is not the really the mathematics that we ourselves love and seek to spend time with. My students deserve better than that.

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