MATHEMATICS FOR TEACHING: AN ANTHROPOLOGICAL APPROACH AND ITS USE IN TEACHER TRAINING

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The question of the kind of knowledge needed by a mathematics teacher has been a subject of study and discussion within the mathematics education community for the past forty years. Shulman (1986) offered an important elaboration with the introduction of the notions of Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK), which challenged the dichotomy between mathematical and pedagogical knowledge. Since then, several mathematics educators have tried to characterise either SMK (*e.g.,* Even, 1990, 1993) or PCK (*e.g.,* Ball, Thames & Phelps, 2008). The issue has also been studied and discussed in international conferences. [1] Some educational researchers have shown the impact of culture on mathematics teaching and learning, and argued that studies of mathematics for teaching require strong theoretical frameworks (Adler & Huillet, 2008).

This article aims to contribute to this reflection. I show how a strong theoretical framework, Chevallard's (1999) anthropological theory of didactics, helps in the analysis of the mathematical knowledge and competencies needed by a mathematics teacher to teach mathematics in a given institution, being critical of the institutional routines. It first presents the main theoretical inputs to this issue, from Shulman's work to more recent works of some mathematics educators. I then analyse these contributions through the lens of Chevallard's theories, concluding that the distinction between some 'purely mathematical knowledge' and mathematical knowledge used in teaching is not appropriate because mathematics does not exist in a vacuum but inside institutions. In teacher education institutions, mathematics should be taught for the intent of teaching. A new description of mathematics for teaching is then presented. This description is based of Chevallard's (2002) six moments of study and is drawn from an analysis of a teacher's work when planning her lessons.

I also report on how this description of *mathematics for teaching* has been used at Eduardo Mondlane University in two courses to help student-teachers to develop their mathematical knowledge oriented for teaching.

SMK and PCK: early steps

Lee Shulman raised the problem of teacher's knowledge for teaching in 1986 (Shulman, 1986, 1987) in connection to the new reform in the USA. He defined three categories for this knowledge: subject matter knowledge (SMK), pedagogical content knowledge (PCK), and curriculum knowledge. I focus on the first two aspects of this knowledge, SMK and PCK.

In his description of SMK, Shulman argues that teachers' content knowledge should not be limited to knowledge of facts and procedures and that it requires an understanding of both the substantive and the syntactic structures of the subject matter:

> The substantive structures are the variety of ways in which the basic concepts and principles of the discipline are organised to incorporate its facts. The syntactic structure of a discipline is the set of ways in which truth or falsehood, validity or invalidity, are established. (1986, p. 9)

This distinction between substantive and syntactic structures of knowledge is very important in mathematics education because in many countries the teaching of mathematics does not take into account these two structures. Facts (definitions, theorems, properties) and procedures (formulas, techniques for solving some kinds of tasks) are usually taught without linking them, and well-established demonstrations are usually given to students. For example, in Mozambican secondary schools, the teaching of limits of functions has usually two main components: a theoretical one, including the ε-δ definition and some proofs using this definition; and a practical one, including the calculation of limits. [2] This dichotomy also exists at university level, including teacher-training programmes. As a consequence, teachers understand *that* something is so, but not *why* it is so – to use Shulman's terms; and this happens not only in developing countries as Mozambique [2], but also in developed countries (see, *e.g.,* Proulx, 2007).

Shulman also highlights the particular relevance of pedagogical content knowledge,

> because it identifies the distinctive body of knowledge for teaching. It represents the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction. (1987, p. 8)

This idea was quite new in the 80s, because content and pedagogy were usually considered separately in teacher training. A teacher was required to have a strong mathematical knowledge of the discipline(s) to be taught and some general pedagogical knowledge, but the idea of blending

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content and pedagogy was not (and still is not) present in many teacher-training courses.

Shulman (1986) describes pedagogical content knowledge as follows:

> Within the category of pedagogical content knowledge, I include, for the most regularly taught topics in one's subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations – in a word, the ways of representing and formulating the subject that makes it comprehensible to others. (1986, p. 9)

It also includes "an understanding of what makes the learning of specific topics easy or difficult" (1986, p. 9).

Ball *et al.* (2008) pointed out that the work of Shulman and his colleagues brings two central contributions to the field: it "reframe[s] the study of teacher knowledge in ways that included direct attention to the role of content in teaching" (p. 395), and it "represent[s] content knowledge as a special technical knowledge key to the profession of teaching" (p. 396). [3]

Shulman's descriptions of the knowledge needed by a teacher have been further developed and used by several mathematics educators, not only in mathematics, but also in other disciplines, such as chemistry (Davidowitz & Rollnick, 2005) and physics (Jita & Ndlalane, 2005). Here I will focus on the studies on mathematics for teaching based on Shulman's ideas.

Even's description of SMK

In line with Shulman, Even (1990, 1993) divides the knowledge that a teacher needs to teach a specific concept in terms of SMK and PCK. She builds an analytical framework for SMK for teaching a specific mathematical topic, which she applies to the study of the concept of functions. Within this framework, seven main facets of a teacher's knowledge of a concept are considered (Even, 1990):

- essential features, described as dealing with students' concept image and paying attention to the essence of the concept;
- different representations, which allow a deeper and more powerful understanding of the concept;
- alternative ways of approaching, which allow the teacher to make a good choice between these alternative approaches;
- the strength of the concept, described as the powerful characteristic of the concept;
- basic repertoire, including powerful examples which allow a deeper understanding of the concept;
- knowledge and understanding of the concept, which includes its connections to other concepts and procedures; and
- knowledge about mathematics, linking to the nature of mathematics.

The analysis of this framework and its application to the concept of limit show that, although this classification points to critical aspects of a mathematics teacher's knowledge, it does not appear to be systematic. [2] Only two categories – the strength of the concept and knowledge about mathematics – strongly refer to SMK. Four other categories refer to both mathematical and pedagogical knowledge, and can be seen to belong to both SMK and PCK: essential features linked to students' concept image; different representations; alternative ways of approaching the concept; and basic repertoire. These categories all refer to teaching practice. The seventh category, knowledge and understanding of the concept, refers to the quality instead of the content of teachers' knowledge.

To conclude, I would say that, within Even's framework, the distinction between SMK and PCK is blurred. This could be because it seems difficult to describe SMK without reference to teaching practices.

Mathematics for teaching

Ball and colleagues introduce the notion of mathematical knowledge for teaching (MKT). Focusing on primary school teachers' practice, these scholars seek to develop a "practice-based theory of mathematical knowledge for teaching" (Ball, Bass & Hill, 2004, p. 55). They ask the following questions:

> What mathematical knowledge is entailed by the work of teaching mathematics?
>
> What and how is mathematical knowledge used in teaching mathematics? How is mathematical knowledge intertwined with other knowledge and sensibilities in the course of that work? (2004, p. 55)

Using Shulman's division between SMK and PCK, they introduce the notion of MKT and distinguish two components of SMK: Common Content Knowledge (CCK) and Specialized Content Knowledge (SCK).

CCK is defined in different ways in different papers:

> "the knowledge that one expects mathematically literate non-teaching adults to hold and also represents the content traditionally taught to middle school students" (Hill, 2007, p. 98);
>
> "the knowledge of the subject a proficient student, banker, or mathematician would have" (Hill, Rowan & Ball, 2005, p. 387);
>
> "the mathematical knowledge and skill used in settings other than teaching" (Ball *et al.,* 2008, p. 32).

In contrast, the corresponding definitions of SCK are similar to one another, describing SCK as the mathematical knowledge that teachers need to conduct their work.

While it seems relatively easy to describe SCK, describing CCK seems to be problematic, as can be seen by the very different explanations given. These explanations all refer to some setting (everyday life, bankers' work, students' work, mathematicians' work), showing the difficulty of defining what we could call 'purely mathematical' knowledge, that is, mathematics out of any practice.

Ball and colleagues (2004) also introduced a new notion for describing mathematics for teaching: the idea of 'unpacking'. They argue that in advanced mathematical work, knowledge is 'compressed' and that teachers need to 'decompress' or 'unpack' this knowledge for their students. I will expand on this important notion later.

Is compressed mathematics (or in Shulman's terms the substantive structure of mathematical knowledge) SMK, CCK or 'purely mathematical knowledge'? Why does this difficulty in describing 'purely mathematical' knowledge exist? Does this knowledge exist? I will examine these questions from an anthropological point of view, using Chevallard's anthropological theory of didactics.

Chevallard's anthropological theory of didactics

Chevallard locates mathematical activity, as well as the activity of studying mathematics, within the set of human activities and social institutions. This model is based on the fact that any human activity can be subsumed as a system of tasks (Bosch & Chevallard, 1999; Chevallard, 1999). Inside a given institution, there are generally one or few techniques to solve a task recognized by the institution.

The institutional relation to an object is shaped by the set of tasks to be performed, using specific techniques by the subjects holding a specific position inside the institution. In an institution, a specific kind of task is usually solved by using a single technique. Each kind of task and the associated technique form the *practical block* (or know-how) of a *mathematical organisation* (MO). Most of the tasks become part of a routine; the task/technique practical blocks appear to be *natural* inside this institution.

However, there is an ecological constraint to the existence of a technique inside an institution: it must appear to be understandable and justified (Bosch & Chevallard, 1999). This is done by the technology, which is a rational discourse to describe and justify the technique. These ecological constraints can sometimes lead to a contradiction, given that students' ability to understand will be constrained by their age and previous knowledge. It can be difficult for a technique to be both understandable and justified at the same time.

The technology itself is justified by a theory, which is a higher level of justification, explanation and production of techniques. Technology and theory constitute the *knowledge block* of a MO. According to Chevallard (1999), the technology-theory block is usually identified with *knowledge [un savoir],* while the task-technique block is considered as *know-how [un savoir-faire].* The two components of an MO are summarized in the figure 1.

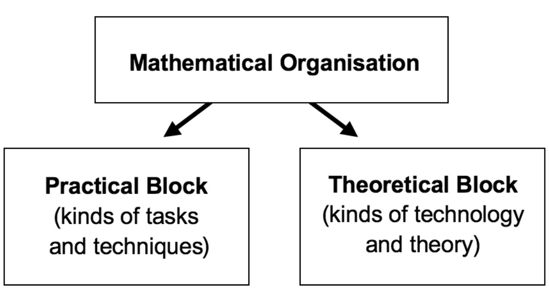

Figure 1. Mathematical organisation

To teach a mathematical organisation, a teacher must build a *didactical organisation* (Chevallard, 2002). To analyse how a didactical organisation allows the set-up of a mathematical organisation, we can first look at the way the different moments of the study of this MO are settled in the classroom. Chevallard (2002) presents a model consisting of the following six moments of study: *first encounter* with the MO, *exploration of the task* and *emergence of the technique,* construction of the *technological-theoretical block, institutionalisation,* work with the MO (particularly *the technique*), and *evaluation*. The order of these moments is not fixed. Depending on the kind of didactical organisation, some of these moments can appear in a different order, but all will probably occur. For example, the study of mathematical organisations at university level is often divided in theoretical classes and tutorials. The theoretical block is presented to students in lectures, as an already produced and organised body of knowledge, and tasks are solved using some techniques (practical block) during tutorials. In this way, there is a disconnection between the theoretical component of the organisation and its applications.

Using Chevallard's approach to analyse MKT

The notion of mathematical organisation

Even's analysis aims to build an analytical framework for teaching a specific *topic* in mathematics. In fact, most of the seven aspects presented as the main facets of teachers' SMK about a topic refer and are applied to a concept, in this case the concept of function. These aspects are: essential features, different representations, alternative ways of approaching a concept, the strength of the concept, and knowledge and understanding of a concept.

Even uses 'basic repertoire' to refer to principles, properties, and theorems, and their applications to several kinds of tasks. Knowledge about mathematics, on the other hand, refers to the nature of mathematics, the ways, means, and processes of production of mathematical knowledge, being "meta mathematics", which transcends a mathematical *concept* and even a mathematical topic. Ball *et al.* (2004) also refer to a *topic:* they begin their paper with the analysis of the specific mathematical topic of multiplication of decimals. They then refer to teachers' mathematical knowledge of the *content*.

The introduction of the notion of mathematical organisation (MO) is helpful from two points of view. First, it helps clarify the focus: *concept* is restrictive because it indicates only one kind of mathematical object; *content* is less restrictive, but not clearly defined. Chevallard (1999) defines a MO as the mathematical reality that can be constructed in a classroom where a specific content is studied. In this way, a MO is located within a specific institution and the same content can give rise to different MOs, depending on the institution where it takes place. It can be very specific (*e.g.,* the multiplication of decimals), less specific (*e.g.,* limits of functions in Mozambican secondary schools), or much more general (*e.g.,* Calculus).

On the other hand the notion of MO re-locates mathematical activities within the core bosom of human activities. It is a special kind of praxeology.

Praxeology

Chevallard (2002) considers that any human activity can be analyzed using only one model, which he calls *praxeology,* because it has two components: a practical block (*praxis*) and a technological-theoretical block (*logos*). For example, for a very simple task such as that of brushing one's teeth, there exist some techniques. Parents usually teach their children one of these techniques, giving explanations such as, "You must brush your teeth and your tongue to remove the bacteria and freshen your breath; otherwise you will have cavities, tartar, and bad breath". This is a technological discourse. This technology can be explained by chemical reactions between bacteria and ivory. Most people do not have a precise knowledge of these chemical reactions, but such knowledge constitutes the theory of this praxeology. Looking at a MO as a praxeology helps us clarify a teacher's tasks when constructing a didactical organisation, and consequently, describe the mathematical knowledge needed to perform these tasks.

A new description of mathematics for teaching

The general task of building a new didactical organisation to teach a MO can be divided into several smaller tasks, according to the different moments of study. [2] The teacher has to:

1. Introduce the mathematical organisation to her students (first encounter with the MO);
2. Introduce some tasks and some techniques to solve these tasks (practical block);
3. Justify and explain these tasks and techniques through a technological discourse (knowledge block);
4. Clarify what students need to know and link this knowledge to the social knowledge (institutionalisation);
5. Organise students' work of the techniques;
6. Evaluate students.

What kind of knowledge does a teacher need to possess to perform these tasks?

First of all, the teacher must choose a suitable way to organise her students' first encounter with the mathematical organisation. Therefore, she needs to know several different ways of doing that; she also needs to know her students' conceptions about this MO and related MOs, as well as the difficulties students usually face when studying it.

Then, to help her students explore the MO, the teacher must also be able to lead them to work with different semiotic representations. She must give students different kinds of tasks and lead them to use different techniques to solve these tasks, choosing a suitable technique for a specific task. This means that she needs to have a good knowledge base of the different semiotic representations in which this concept can be studied, an extended basic repertoire of tasks within these representations, and to the capability of shifting from one representation to another.

Furthermore, the teacher has to choose the technological elements she will give her students, in order to justify and explain the techniques introduced to solve the tasks. Which concept definition(s) should students be given, according to their age and previous knowledge? Which theorems, which proofs can justify these rules? Are students able to understand these proofs? If not, how can these rules be explained? Can a shift of semiotic representation help explain these rules? Here again the teacher needs good knowledge of the MO to be taught, but also knowledge of different representations and students' previous knowledge.

The analysis of the knowledge needed by a teacher to set up a didactical organisation to teach a specific MO led to the following categories of mathematics for teaching (MfT):

1. Strong mathematical knowledge of the MO taught within the institution; this includes definitions of concepts, theorems and their proofs, correct use of notations and symbols, as well as general knowledge about mathematics.
2. Knowledge about the social justification to teach this MO: why is this MO currently taught in this institution? This includes its relations to other MOs, and its applications to everyday life, in mathematics, or in other disciplines.
3. Knowledge about organising students' first encounter with this MO.
4. Knowledge about the practical block of the MO (tasks and techniques); this includes a repertoire of different tasks using different representations, or the ability to shift from one representation to another, and different technologies to solve these kinds of tasks.
5. Knowledge on about constructing the knowledge block (technological elements to justify the techniques) according to learners' age and previous knowledge.
6. Knowledge about students' conceptions and difficulties when studying this concept.

This description of MfT does not distinguish between mathematical and pedagogical knowledge; rather it considers that each of these aspects of MfT have the two components merged together, although some of these aspects are more mathematical and others more pedagogical.

Relating to other studies

The above presentation of mathematics for teaching relates to various descriptions provided by other researchers.

Most of Even's categories of SMK can be identified in this new presentation of MfT. For example: the first encounter with a MO includes the way of approaching a concept; basic repertoire and different representations are included in the construction of the practical block; the strength of the concept is part of the social justification for teaching this concept. However, this analysis of MfT, which is based on teachers' actions to help students perform different mathematical tasks, structures Even's categories. In fact, Even's categories can be seen as parts of this new framework.

Ball and colleagues provide a list of critical tasks that teachers have to tackle in their classroom practice to 'unpack' the mathematical knowledge. Presented below are some examples of how these tasks relate to Chevallard's theory. They (2004) contend that, when confronted novel or non conventional student solutions, teachers must be able to analyse these methods and decide whether they work, not only for the specific task under consideration, but for all cases belonging to this kind of task. This relates to knowledge of several techniques to solve a task, and knowledge of the technologies that explain these techniques.

These scholars also assert that teachers must be able to select definitions that are mathematically appropriate. An additional criterion for a 'good' mathematical definition, according to these scholars, is "whether or not [the definition] is usable by pupils at a particular level" (2004, p. 58). In that case, the content is a concept, and the teacher must be able to select a definition for students. Ball *et al.* (2004) mention two constraints: the definition must be both correct and usable. These constraints relate to the two constraints identified by Chevallard (1999), not only for definitions, but for all parts of the theoretical block of a MO. (*e.g.,* for demonstrations of algorithms, properties, and theorems). Chevallard argues that the theoretical block must be both mathematically acceptable and understandable by students of a particular level.

As an example, Ball *et al.* (2004) analyse the multiplication of two decimals 1.3 × 2.7 (kind of task). Let's look at this example through the lens of the anthropological theory of didactics. This task is solved using an algorithm: find the product of 13 × 27, and then, starting from the right end of the product, move two digits on the left, and place the decimal point. In Chevallard's terms, this is the technique associated to this kind of task. Ball *et al.* (2004) then emphasize that teachers should be able to analyse students' wrong answers, explain why they are wrong, and use different representations to represent the meaning of the algorithm. These explanations and representations, which aim to explain the algorithm and its possible deviations, belong to the level of the technology. This article does not elaborate on the level of the theory, which in this case could be deep knowledge of the decimal system needed to explain why this algorithm works. In fact, this theoretical level is not understandable to primary school students, and, therefore, it is not the focus of this writing.

In practice, mathematics teachers know the algorithm to perform multiplications of decimals (practical block); they might also have studied the decimal system in advanced mathematical courses they had taken during their training (theory). [4] However, a link is missing: how to use the theory in order to produce an explanation at the technological level, which is mathematically acceptable and understandable to students? This is what Ball *et al.* call 'unpacking', and that, in Chevallard's terms, could be called the technological discourse.

Two other aspects of teachers' work presented by Ball *et al.* (2004) can be analysed in terms of the knowledge of the technological part of the MO. These are: teachers must be able to 'unpack' mathematical ideas through explanations and use of different representations; and teachers must help students connect ideas, build links and coherence in their knowledge. Being able to produce a technological discourse would enable teachers to explain the procedures to their students, but also to understand and explain student errors and to analyse other methods of solving a kind of task. What Ball and colleagues put forward with the notion of 'unpacking' can be seen as a deep analysis of what could be seen as teachers' technological discourse in primary schools.

How could these teacher competences be developed during teacher training? It is this challenge I faced in the teacher-training programme at Eduardo Mondlane University (UEM).

Training teachers to develop competences

In the teacher-training programme at UEM, student-teachers have to take three years of mathematics courses, together with students studying 'pure mathematics'. During their fourth year, they choose between a 'pure mathematics' specialization and a teacher education specialization. I am in charge of two teacher education specialization courses: Didactical Analysis (7th semester of the programme [5]) and Didactical Engineering (8th semester [6]). These two courses constitute the basic professional training of prospective mathematics teachers. They are complemented by two other courses, Teaching of Geometry, and Technological Tools for Teaching; student-teachers can enrol in these courses during their 8th semester. Student-teachers are then supposed to apply their knowledge and competences in practice (also during the 8th semester).

The main objectives of the courses I teach are that student-teachers will be able to analyse mathematics teaching in terms of content and methods (Didactic Analysis) and to plan the teaching of a mathematical organisation for Mozambican secondary school (Didactical Engineering). A key challenge I face is how to organize these two courses for prospective teachers so that these teachers link their mathematical knowledge to their pedagogical knowledge in order to improve exisiting teaching practices in schools. This has been done according to the description of MfT presented above.

The first activity within the Didactical Analysis Course has been the analysis of first year university students' answers to a problematic mathematical task. [7] This is a way of showing prospective teachers the importance of didactical analysis, and how a rote teaching of procedures can lead to learning without understanding. It is also a way of putting them in the teacher's position to assess real student productions. Other didactical texts (*e.g.,* secondary school syllabi, final examinations' tasks, textbooks, worksheets) are then analyzed, taking into account the need for conceptual teaching. This is done through a few short presentations by the lecturer and discussion of some theoretical texts, but mainly through in-class group presentations and discussions. I also use videos of different kinds of mathematics classes; during class, student-teachers observe these videos and are asked to analyse several ways of organising teacher's and students' activities in the classroom.

In parallel with the work done in class, each student has to undertake a full didactical analysis of a MO (or part of a MO) from the upper secondary school syllabus (between 10 to 12 lessons). This analysis includes:

- critical analysis of the syllabus referring to this MO;
- critical analysis of related MOs in previous grades (required knowledge to study the MO) and subsequent grades (where and how this knowledge will be used by students in the future);
- critical analysis of this MO in Mozambican textbooks;
- analysis of tasks related to this MO in secondary school final examinations;
- critical analysis of this MO in at least two textbooks of other countries;
- analysis of main student mistakes when studying this MO;
- proposals on how to improve the teaching of this MO in Mozambican secondary schools.

The MOs are chosen by the student-teachers at the beginning of the semester from a list presented by the lecturer. Student-teachers have to work on their MO throughout the semester, using the knowledge and competences developed during the course. Student-teachers have periodic individual sessions with the lecturer to support their work, and periodic presentations of part of their work to their colleagues. Student-teachers' final evaluation is based on their portfolio and their Power Point presentation to the class.

The Didactical Engineering course is organised in similar ways: in this course, student-teachers' individual portfolio consists of a full plan for teaching an upper secondary school MO (between 10 to 12 lessons), including all moments of the study of a MO, as presented in the previous sections.

Student-teacher difficulties

Student-teachers usually perform quite well in analyzing the didactical materials as required by the first course (Didactical Analysis), but they face many difficulties producing new materials (for the Didactical Engineering course). They usually have difficulties understanding the 'in situ-encounter' [8] of a mathematical object and organizing activities leading students to discovery. Student-teachers' lessons are usually based on teacher presentation of the content; these lessons hardly introduce any proof for given theorems, or any technological elements to justify the techniques. In these lessons, students are mainly required to imitate the teacher, without understanding.

Student-teachers attribute these difficulties to their lack of experience with this kind of approach in mathematics classes. In Mozambique, mathematics is usually taught in a very traditional way: the teacher explains the content to her students and shows them how to solve several kinds of tasks. In secondary schools, theorems are hardly proved. At the university level, students usually learn proofs but do not learn *how* to prove. As a consequence, they memorize theorems and techniques in a very rote way. Because they learn 'compressed' mathematics and they are not used to reflect on their learning, it is very difficult for them to 'unpack' the mathematics they have learnt.

Concluding remarks

This paper has analyzed how the mathematical knowledge needed by a teacher has been described by several mathematics educators; all of whom have grounded their analysis in Shulman's argument that a mathematics teacher needs to develop a mathematical knowledge in ways that are appropriate for teaching. In this paper, the mathematical knowledge needed by the teacher has been analyzed through the lens of Chevallard's anthropological theory of didactics, which looks at a teacher's task as the construction of a didactical organisation aimed at teaching a specific mathematical organisation to specific students inside a specific institution.

I have argued that the distinction between 'purely mathematical' knowledge (SMK or CCK) and mathematical knowledge adapted for teaching (PCK or CCK) is not appropriate. Mathematics does not exist 'in a vacuum'; it lives inside institutions, and it must live in specific ways in teacher-training institutions. I have then presented a new description of mathematics for teaching (MfT) based on Chevallard's theories and on the results of my research.

In the second part of this paper, I have shown how this description of MfT has been used to define the competences that prospective teachers need to develop and consequently to design two courses of a teacher-training programme. I have also briefly explained some of the difficulties encountered by prospective teachers enrolled in these courses. These difficulties can be explained by student-teachers' prior experiences of mathematics as learners of the subject. Several studies showed that teachers tend to teach in the same way they have been taught. In Mozambique, mathematics is taught in a very traditional way, and students are used to memorizing definitions, theorems, proofs, and techniques to solve some kinds of tasks, even if they don't understand them.

In teacher training, we should ask ourselves the following questions: What kind of mathematics teachers do we want to train? Do we want to train teachers who will reproduce the present system, by teaching their students algorithmic techniques to solve some kinds of tasks, without attending to the understanding of concepts and without helping their students to reflect on mathematical methods of working or to apply their knowledge to everyday practice or to other sciences? Or do we want to train teachers who will be active in trying to change the way mathematics is taught, in terms of contents, methods, and means, teachers who will seek to teach for more conceptual understanding and who will be able to develop their students' competences?

If we want to train teachers who will be able to teach in a more active and creative way, we must give them the opportunity to experience this kind of teaching as students. I suggest that, in teacher-training programmes, mathematics courses be taught in a way that allows student-teachers to 'unpack' the mathematical knowledge, apply this knowledge to solve real problems from everyday life or from other sciences, using as much as possible active methods of discovery. This should be done since student-teachers' first year of training. It is *our* challenge, not only at Eduardo Mondlane University, but in all teacher-training institutions.

Notes

[1] For example, at the 11th International Congress on Mathematical Education in July 2008, the Topic Study Group 27: "Mathematical knowledge for teaching" organised by Deborah Ball and Jill Adler.

[2] See D. Huillet (2007) *Evolution, through participation in a research group, of Mozambican secondary school teachers' personal relation to limits of functions,* unpublished Ph.D. dissertation, Johannesburg, ZA, University of the Witwatersrand.

[3] For a detailed description of Shulman's notions of SMK and PCK and further developments, see Ball *et al.* (2008).

[4] Many primary school teachers do not have mathematical training and, consequently, do not reach the level of being able to understand (and provide) theoretical explanations for some techniques.

[5] Analysis of mathematics specific teaching materials

[6] Production of mathematics specific teaching materials.

[7] I use the expression 'problematic' to indicate that the task is not solved by using only a rote procedure, but requires some interpretation.

[8] In the 'in-situ encounter', the student, alone or with a group, is confronted with a task where the object at stake is expected to appear as necessary to answer one or more specific questions, while in the 'cultural-mimetic problematic', the new object of knowledge is presented as already existing in some social practice (Chevallard, 2002).

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Are *tangents* (*i.e.,* tangent lines) from calculus related in any way to *tangent* (a ratio of side lengths) from trigonometry? If not, why do they share the same name?

(submitted by Moshe Renert; suggested by Aaron Renert)
