Instruments for Semiotic Mediation in Primary School Classrooms [1]

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Instruments have been used for centuries in mathematical experience and in the teaching tradition as well We cite several examples:

1) concrete materials, artificially designed by educators to reflect underlying mathematical ideas (e.g. Dienes blocks) that date from the nineteen-sixties and were (and still are) very popular among teachers;

2) cultural instruments inherited from tradition (e.g rulers, abaci, compasses, curve-drawing devices) that have followed or anticipated the theoretical development of mathematics;

3) technological objects taken from everyday life (e.g. scales, gears, coins) that bear witness, in their functioning, to hidden pieces of mathematics knowledge;

4) software developed by information technology (e.g. CAS or dynamic geometry systems) that allow a quick approach and solution to complex problems.

The first and the last case are meaningful: teaching aids in the past and new software today have often been advertised as useful, powerful and effective instruments for solving the problems of teaching. Expectations have been often frustrated, however. Research in didactics of mathematics has shown that:

> artefacts become efficient, relevant and transparent through their use in specific activities, in the context of specific types of social interactions, and in relation to the transformations that they undergo in the hands of users. (Meira, 1995, p. 110)

In this article, we shall analyse two cases of instruments (the compass and the abacus) from type 2 above, which are very commonly found in primary school classes. We do so by inserting them in a Vygotskian framework which allows us to be more precise about the quality of social interactions (individual and group tasks; discussions orchestrated by the teacher), realised under the teacher's guidance, used in order to foster the individual construction of mathematical meanings. The two examples concern two artefacts inherited from tradition and describe the shift from the 'concrete' instrument to the 'mental' one in primary school The users are young pupils, chosen to emphasise the need of starting quite early to nurture a theoretical attitude towards mathematics

1. From the concrete to the mental compass

This activity takes place in a fifth-grade classroom. The classroom is taking part in a long-term teaching experiment from first grade, concerning the modelling of gears The project has two aspects: the (algebraic) modelling of functions (Bartolini Bussi *et al.*, 1999) and the (geometrical) modelling of shapes (Bartolini Bussi *et al*, to appear). For the pupils, (toothed) wheels have been naturally modelled by circles and wheels connected with each other as gears have been modelled as tangent circles (the evocation of wheels appears also in the protocols) They have discovered by experiment and transformed some elementary properties of tangent circles into fundamental statements, such as the alignment of the two centres and the point of tangency and the related relation between the distance of centres and the sum of the radii.

The pupils are capable of discussing effectively with each other, as their teacher (Mara Boni) is a member of the research team on *Mathematical Discussion,* which analyses the limits and advantages of different types of discussion orchestrated by the teacher (Bartolini Bussi, 1996; 1998a; 1998b) and has introduced mathematical discussion into that classroom since the first grade. [2] The pupils are also accustomed to producing detailed individual written protocols, with a clear explanations of their processes: this custom too is the outcome of a very special classroom culture, where individual tasks and discussions on the individual strategies are systematically interlaced with each other

The pupils are acquainted with the use of the compass to draw circles more precisely than freehand. During the second and third grade, they have worked for some sessions on the compass, first trying to invent their own instruments to draw circles and then appropriating the existing instrument 'compass' and the manual procedure (not so easy for young pupils) by which to use it effectively. They have often used the compass to produce round shapes, also in art lessons with the same teacher (to imitate some drawings by Kandinsky).

The individual problem

The pupils have been given the following individual problem on an A4 sheet of paper.

> Draw a circle, with radius 4cm, tangent to both circles
>
> Explain carefully your method and justify it

(On the sheet, the drawn circles have radii of 3cm and 2cm and the distance between their centres is 7cm. See Figure 1.)

All the pupils of the classroom have produced a solution by trial and error, adjusting a compass to produce a circle

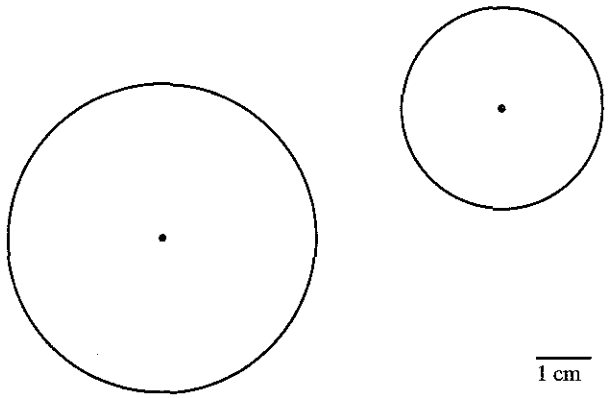

Figure 1:

that looks tangent to both. Some of them have found two solutions (symmetrically placed) The teacher (Mara Boni) collects all the individual solutions, analyses them and selects Veronica's solution (below) for further activities

> The first thing I have done was to find the centre of the wheel C;
>
> I have made by trial and error, in fact I have immediately found the distance between the wheel B and C. Then I have found the distance between A and C and I have given the right 'inclination' to the two segments, so that the radius of C measured 4cm in all the cases. Then I have traced the circle.
>
> JUSTIFICATION
>
> I am sure that my method works because it agrees with the three theories we have found:

I) the points of tangency G and H are aligned with ST and TR;

II) the segments ST and TR meet the points of tangency G and H;

III) the segments ST and TR are equal to the sum of the radii SG and GT, TH and HR.

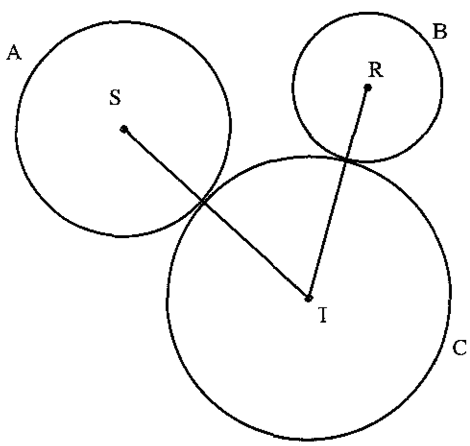

Figure 2:

The collective discussion

A week later, the teacher gives all the pupils a copy of Veronica's solution. Then she introduces the topic of discussion.

Teacher: Veronica has tried to give the right inclination. Which segments is she speaking of? Many of you open the compass by 4 cm. Does Veronica use the segment of 4 cm? What does she say she is using?

[Veronica's text is read again]

Jessica: She uses the two segments ..

Maddalena: .. given by the sum of radii

Teacher: How did she make?

Giuseppe: She has rotated a segment

Veronica: Had I used one segment, I could have used the compass

[Some pupils point with thumb-and-index fingers at the segments on Veronica's drawing and try to 'move' them. They pick up an ideal segment as if it were a stick and try to move it]

Francesca B.: From the circle B have you thought or drawn the sum?

Veronica: I have drawn it.

Giuseppe: Where?

Veronica: I have planned to make RI perpendicular [to the base side of the sheet] and then I have moved ST and RT until they touched each other and the radius of C was 4 cm

Alessio: I had planned to take two compasses, to open them 7 and 6 and to look whether they found the centre. But I could not use two compasses.

Stefania P.: Like me; I too had two compasses in mind.

Veronica: I remember now: I too have worked with the two segments in this way, but I could not on the sheet.

[All the pupils 'pick up' the segments on Veronica's drawing with thumb-and-index fingers of the two hands and start to rotate them. The shared experience is strong enough to capture all the pupils.]

Elisabetta: [excited] She has taken the two segments of 6 and 7, has kept the centre still and has rotated: ah, I have understood!

Stefania P.:	. to find the centre of the wheel ...
Elisabetta:	after having found the two segments ...
Stefania P.:	... she has moved the two segments
Teacher:	Moved? Is 'moved' a right word?
Voices:	Rotated ... as if she had the compass.
Alessio:	Had she translated them, she had moved the centre
Andrea:	I have understood, teacher, I have understood really, look at me . .

[The pupils continue to rotate the segments picked up with hands.]

Voices:	Yes, the centre comes out there, it's true.
Alessio:	It's true, but you cannot use two compasses .
Veronica:	. . you can use first on one side and then on the other.
Teacher:	Good, pupils. Now draw the two circles on your sheet.

[All the pupils draw the two circles on their sheet and correctly identify the two possible solutions for the centres, see Figure 3.]

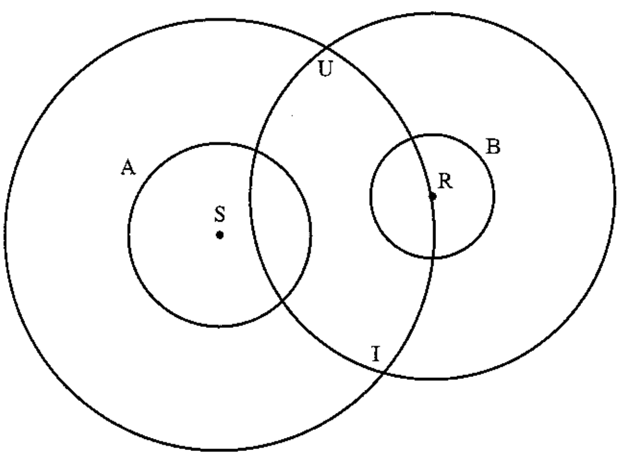

Figure 3:

Comments

In this episode, we are real-time observing the emergence of an enriched use of the compass, one that is entering the classroom culture. The compass is going to be used not to produce a fair round shape, but to find a point (or better, two points) that are at a given distance from two given points. The way of using the compass (i.e. the gesture of handling and tracing the curve) is the same when a pupil wishes to produce a round shape and when s/he wishes to find a point at a given distance, but the senses given by the pupils to the processes (gestures) and to the products (drawings) are, in the second case, very different. The pupils are producing a problem-solving strategy that:

(i) can be used in any situation;

(ii) can produce and justify the conditions of possibility in the general case;

(iii) can be defended by argumentation referring to the accepted theory.

We shall reconsider this point later.

The geometric compass, embodied by the metal tool stored in every school-case, is no longer only a material object: it has become a mental object, whose use may be substituted or evoked by a body gesture (rotating hands or arms) or even by the product of the gesture, i.e. the drawn curve. Even if the link with bodily experience is not cut (rather, it is emphasized), the loss of materiality allows a distance to be taken from the empirical facts, transforming empirical evidence of the drawing that represents a solution (whatever the early way of producing it) into the external representation of a mental process. The (geometrical) circle is not an abstraction from the perception of round shapes, but the reconstruction, by memory, of a variety of acts of spatial experience (a 'library' of trajectories and gestures, see Longo, 1997).

In the above episode, we observed the integration of two ways of thinking of circles. Recalling the history of geometry, it is the integration of the mechanical/dynamic/procedural approach of Hero ('a circle is the figure described when a straight line, always remaining in one plane, moves about one extremity as a fixed point until it returns to its first position') with the geometrical/static/relational approach of Euclid ('a circle is a plane figure contained by one line such that all the straight lines falling upon it from one point among those lying within the figure are equal to one another'). In this case, the standard compass is only the prototype of a larger class of instruments (drawing instruments) which were used for centuries to prove the existence of and to construct the solutions of geometrical problems and of algebraic equations as well (Lebesgue, 1950). The experience of the continuity of motion was used in place of the theoretical foundation of mathematical continuity which was still lacking at that time.

Hence, exploring the relationships between these two ways of thinking of circles and the role played by the compass is an epistemologically correct way to approach the problem of the continuum very early on. Even if the flow of the discussion is natural and fluent, the process is not spontaneous at all: the care of the teacher in choosing an effective protocol and in encouraging the use of gestures is evident, as is the interaction between pupils with a deep exploration of the mental processes.

2. From the 'concrete' to the 'mental' abacus

After revisiting the standard compass, we illustrate a second example concerning the abacus. If the compass (and other drawing instruments) brings into focus the problem of the continuum, the abacus focuses on the polynomial representation of natural numbers. In this case, we draw on the outcomes of a very recent teaching experiment carried out in the first and the second grade with different pupils but the same teacher (Mara Boni)

The abacus had been introduced at the end of the first grade to count and to reckon with. The children handled the abacus under the teacher's guidance. They were accustomed to representing the abacus with a typical sketch, where the drawing of the rods for the marbles was accompanied by the representation of symbols for units, tens, and so on.

The scheme of the teaching experiment in the second grade was the following:

1) initial individual problem;

2) collective discussion about the problem;

3) meta-discussion with individual tasks;

4) final individual problem.

The initial individual problem

The starting point was the following individual problem

> In our classroom, each pupil is building his or her own individual abacus. A child from another classroom has seen us working and has asked: "What is the abacus? What is it for? How do you use it?". Try to answer these questions.

All the pupils answered the questions. Anna gave the following answer (an excerpt):

> The abacus is a box with 4 rods and some marbles to decipher the number [See Figure 4.]

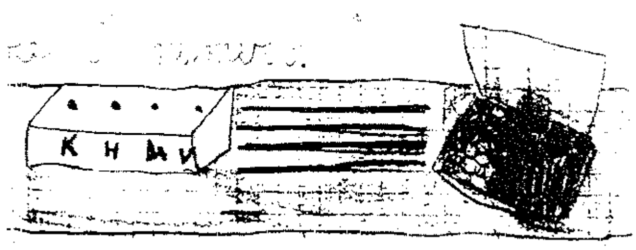

Figure 4.

The collective discussion

The teacher's attention has been captured, in Anna's protocol, by the contrast between the concrete description of the artefact biased by the activity of constructing it and the apparently abstract description of the goal *(decipher the number)*. She chooses Anna's answer to start the discussion. Her hope is to shift the pupils' attention towards the conventions used to represent numbers in base ten.

Teacher: Today, I'd like to discuss with you an interesting thing. In the problem, when you explained what an abacus is and what it is for, you said that it is a thing, a tool, an object for reckoning and for exchanging [ten units for a ten]. You all have said it. One girl, Anna, said: "the abacus has some marbles to decipher the number". She used the word 'decipher'. What did she mean?

Anna: I meant that a marble on the abacus is for deciphering how the number must be written.

Voices: ... I do not understand ...

Anna: Using marbles we can decipher well the number, i.e. we put the marbles to understand well the number, and it is for deciphering the marbles and we can write the number.

The discussion goes on for a while (53 utterances) under the teacher's guidance, yet these are the only interventions of Anna's in the discussion. Anna's utterances do not add anything new. The class constructs an example of representation for the number 18. Many pupils (Anna is silent) take part in the discussion and succeed in overcoming the conflict between the existence of infinitely many numbers and the limit of the abacus as a concrete artefact.

The meta-discussion

The teacher introduces another form of discussion (a meta-discussion, i.e. a discussion about the previous discussion, see Bartolini Bussi, 1998b). Each pupil is given a copy of the transcript of the previous discussion. The text has been cut into pieces and spaces for individual drawings and comments are left after the utterances which are, in the teacher's analysis, provocative for pupils' reflection. The text is read again aloud, collectively, without introducing further comments. This activity fosters individual representation of the meanings emerging from the dialogue.

In the following, we only present some of Anna's drawings, which show the progressive shift from the concrete abacus towards a mental abacus useful for mental experiments

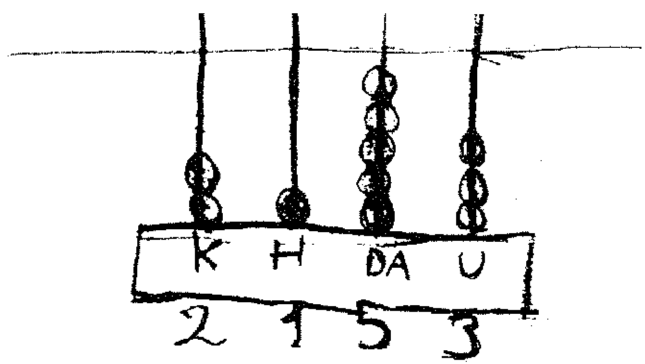

Figure 5.

This is the drawing produced by Anna at the beginning of the meta-discussion, to explain the meaning of *deciphering*. It is the representation of a concrete object, laying on a table. The representation is static, with no reference to gestures The attention is shifted to the function of the artefact. A bit later, Anna succeeds in representing the number 18

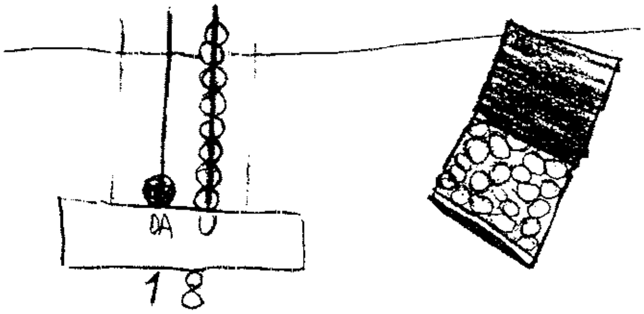

Figure 6:

There is still the table and the concrete object. However, Anna can use it in a skillful and flexible way. She uses the central rods and (later) erases the first and the last one (as if she had pulled rods out of the base), when she notices the mistake

When, a bit later on in the discussion, attention is focused on the fact of infinitely many numbers, Anna introduces another different and personal representation, referring to the tear-off calendar:

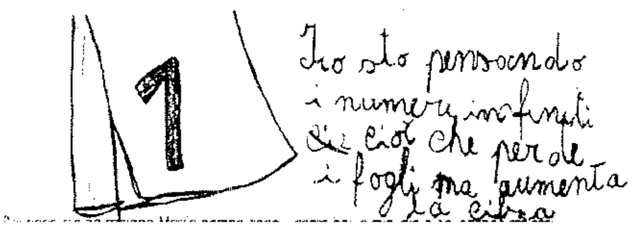

Figure 7:

Anna writes:

> I think of the infinitely many numbers, i.e it loses sheets but increases the number.

Figure 8:

She shows herself able to master the apparent conflict between losing sheets (decreasing) and increasing value.

When the pupils claim that, although the abacus is limited *it is possible to go on in the mind,* Anna draws a balloon, like in a strip cartoon. The abaci are sketched but the base is still very bulky and heavy. (See Figure 8.) The possibility of adding more and more rods, *up to infinity* as a pupil says, fosters another representation. The concrete object is transformed. In the mind, Anna can use many hands: the left and the right ones are pulling the base to overcome the material stiffness, whilst other pairs of hands are putting marbles in the rods. The process is virtually iterated by the left arrow.

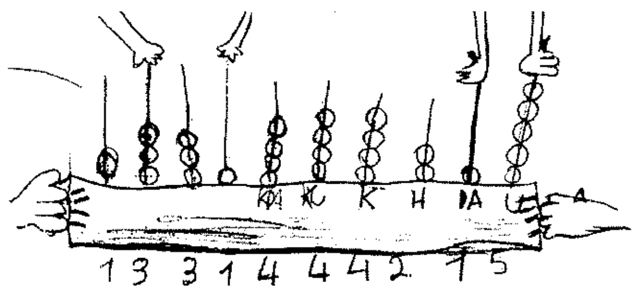

Figure 9:

At the end of the discussion, the final drawing of Anna's is the following, together with a written comment:

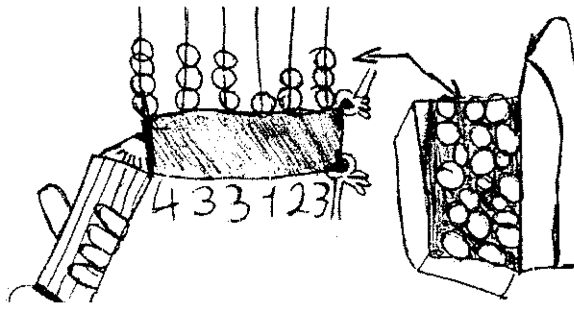

Figure 10:

> The abacus is heavy and cannot stand by itself in the mind. I have created two imaginary hands to keep it still. The drawing hand is mine. I invent the marbles and my hand goes on drawing a longer and longer abacus up to infinity

The recollection of the concrete object is present in the weight and in the box for marbles. The big pencil emphasizes the process of graphical construction, that substitutes for the gestures of putting the marbles on the rods.

The final individual problem

Some days later the following problem is given

> On the back of sheet A, there is a number with 2 digits whilst on the back of sheet B there is a number with 3 digits Can you decide which is the bigger without reading them? Explain well your reasoning

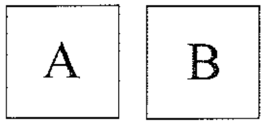

Figure 11:

Anna masters the situation quite well Below is an excerpt from her solution.

> Maybe on the back of sheet A the number is 20 and on B is 122 Yet it might be 10 on A and 200 on B.

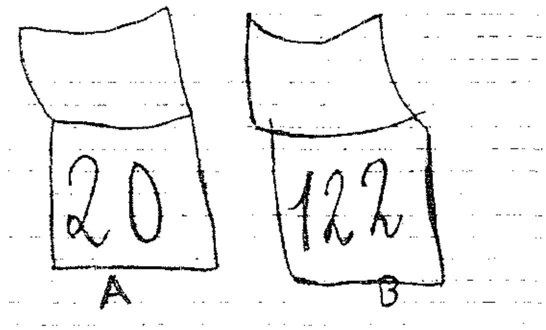

Figure 12:

> The bigger is 122 i.e B, because B has 3 digits and A 2

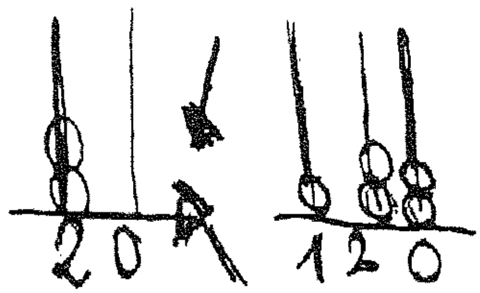

Figure 13:

> In 20, a position is missing (the hundreds). Otherwise they could be equal. Yet B is bigger than A.

Figure 14:

> Until the tens don't reach 99, there is a difference.

Three different representations are used:

- the tear-off calendar;
- the abacus;
- a new representation focused on the positions only.

Comments

In this experiment, we are real-time observing the emergence of the mastery of the conventional notation for numerals by means of the abacus. This use is richer than the one documented in history for centuries and typical of young pupils too at the beginning of their school experience The abacus is going to be used not only to count and reckon with, but also to support and understand the polynomial representation of numbers. The way of using the abacus (by putting marbles onto rods) is similar, but the senses given by the pupils to the processes (gestures) and to the products (written representations and drawings) are, in the second case, very different The pupils are producing a mental tool that can be applied in situations out of reach of real experience (extending its use to infinitely many numbers)

The abacus, a bulky and heavy box with rods and marbles at the outset, is becoming a mental object, whose use may be substituted or evoked by a written representation of numbers focused on the positions only. Even if the link with bodily experience is not cut (rather, it is emphasized), the loss of materiality allows a distance to be taken from the empirical facts: in some phases of the process, the concrete abacus is still in the foreground, but there is a progressive detachment from this concreteness. We are so lucky to have all the 'stills' of Anna's slow detachment from the concrete object. Anna is expected to be able to solve arithmetical problems without the abacus, with only paper and pencil The transition from abacus to paper (with the need to introduce the special sign 0 for the empty groove) is:

> to shift from a *gestural* medium (in which physical movements are given ostensively and transiently in relation to an external apparatus) to a *graphic* medium (in which permanent signs, having their origin in these movements are subject to a syntax given independently of any physical interpretation). (Rotman, 1987, p. 13)

Owing to its nature, the graphic medium will gain autonomy and be used anew without any reference to the abacus in the solution of new problems; moreover, it will also be used in the planning phase, to devise a strategy to attack some arithmetical problems

3. A tentative theoretical framework

The two examples - the compass and the abacus, inserted into suitable classroom practices - are relevant cases of psychological tools or *tools of semiotic mediation*, as Vygotsky (1978) termed them. It is useful to recall here three basic constructs of the Vygotskian approach:

- the zone of proximal development;
- internalisation;
- semiotic mediation

The zone of proximal development is a metaphorical place, one where problem solving is situated: the activity of problem solving is carried on in co-operation between a child and an adult or a more clever peer. With this help, the child may succeed in solving problems that could not be solved by her or him alone This help may be offered in different ways, with the introduction of artefacts, or with the explicit requirement of using a system of signs (natural language, gestures, drawings, conventional systems of representations) The artefacts and the systems of signs share some common features:

- they are beautiful products of a social activity of mankind;
- they are not limited to action on the external world, but rather influence the mental processes of the subject who is using them during the social practice;
- they potentially incorporate mathematical meanings (such as the dynamic construction of circles or the convention of number notation)

In the second example, the exploration of the zone of proximal development is evident in the meta-discussion. Each pupil is encouraged and supported to develop germs of individual processes and to explore the 'new', starting from the recollection of a shared social practice (i e the first discussion and its transcript, as prepared by the teacher) The mathematical meanings are potentially contained in the artefacts, although there is an opaque representation, one that comes out of the shadow only by means of guided classroom practice

This very last feature plays the biggest role in the process of semiotic mediation. In the problem of finding a point at a given distance from two given points (compass) or in the problem of deciding which is the bigger of two numbers, the child draws (to use the Vygotskian word) an instrument into the process (together with all the rules that have been learnt in social interaction), which in this way inhibits a 'naive' answer, one based on previous knowledge and automatisms. The very presence of this instrument constructs a new situation that is richer and richer, when the exploration of the instrument is guided, deep and careful. The joint activity of the teacher and the child surely fosters the child's imitation of the adult's way of acting Yet this imitation of complex action requires the understanding of the other's action, carried out during intense verbal exchanges

Internalisation is the basic mechanism, through which an external process of social interaction (interpsychological) becomes internal (intrapsychological), with an important transformation of its structure. When an external activity shared between an adult and a child is transferred to become internal, it acquires greater freedom: it may be evoked (or 'drawn' into the solution of a problem) and pointed towards directions that are impossible in the concrete world

When the compass is used to produce round shapes, when the abacus is used to count and reckon with, it is mainly oriented toward the outside; when the compass is used to find the points which satisfy a given relationship, when the abacus is used to master the polynomial notation of numbers, they become instruments of semiotic mediation (Vygotsky, 1978), that draw systems of signs into the activity and can control - from the outside - the pupil's process of solution of a problem.

Semiotic mediation is started in the collective phases (discussions orchestrated by the teacher, either after or before the individual task), with a strong emphasis on imitation of gestures and words. The teacher's role in guiding the whole process is essential. Hence, in spite of the big difference between the two artefacts, strong and deep similarities can be envisaged in the management of school activity. They can be summed up by three key terms: social interaction, teacher guidance and imitation. The resulting processes draw on a careful *a priori* analysis of the potentialities of the task towards the development of the higher psychological process, required by this approach to theoretical knowledge

4. Concluding remarks

In this article, we have discussed two examples which are provocative for both teachers and researchers. They concern activity with concrete objects, yet the goal is to transform these objects into mental ones, to be used in mental experiments aimed at approaching some mathematical ideas that are out of reach of empirical thinking (the continuum and the polynomial representation of infinitely many numbers) They are related to the development of a theoretical attitude towards mathematical objects

We claim that the process of building a theoretical attitude towards mathematics is quite long and can take years In our framework, this process is developed from the very beginning of primary school, under the guidance of a cultured adult (the teacher), who, on the one hand, selects the tasks and, on the other, orchestrates the social interaction before or after the individual solution For the learners, gaining a theoretical attitude does not cut the link with concrete (and bodily) experience, but rather gives a new sense to 'the same' concrete experience.

From a research perspective, this set of studies opens a lot of interesting questions For instance, they concern:

- the analysis of distinctive features of theoretical knowledge, at least when the didactical purpose is in the foreground;
- the listing of a larger and larger set of artefacts analysed as tools of semiotic mediation and the study of the effective introduction of these artefacts in the classroom;
- the study of relationships between individual and collective processes in selected cases

Several other case studies are in progress, concerning very intrusive concrete objects (e.g. perspectographs towards the projective extension of the Euclidean plane and the roots of projective geometry; see Bartolini Bussi, Mariotti and Ferri, in press)

Following the same trend, there is also the recent development of 'virtual' microworlds: we may quote the approaches developed by Mariotti (and co-workers) in the *Cabri-Géomètre* setting (Mariotti, 2001) and in *L'Algebrista,* a symbolic manipulator created within *Mathematica* to introduce pupils to algebraic theory (Cerulli and Mariotti, 2001). By means of this wide range of experimental research, we aim to produce a fully-fledged theoretical framework that can produce analytical tools for designing and studying classroom experiments, where recourse to concrete artefacts is linked, right from the early school years, to the aim of nurturing a theoretical attitude towards mathematics.

Notes

[1] This text is related to some lectures given in different places, in Canada (a plenary talk at the Canadian Mathematics Education Study Group, held in Montreal in 2000; see Bartolini Bussi, 2001) and in Italy (national conferences for mathematics teachers)

[2] In Italy, most primary school classes stay together with the same teacher for five years

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